Variable Rare Disasters: An Exactly Solved Framework for Ten Puzzles in Macro-Finance

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Abstract

This paper incorporates a time-varying severity of disasters in the hypothesis proposed by Rietz (1988) and Barro (2006) that risk premia result from the possibility of rare large disasters. During a disaster an asset's fundamental value falls by a time-varying amount. This in turn generates time-varying risk premia and thus volatile asset prices and return predictability. Using the recent technique of linearity-generating processes, the model is tractable and all prices are exactly solved in closed form. In this paper's "variable rare disasters" framework, the following empirical regularities can be understood qualitatively: (i) equity premium puzzle; (ii) risk-free rate-puzzle; (iii) excess volatility puzzle; (iv) predictability of aggregate stock market returns with price-dividend ratios; (v) value premium; (vi) often greater explanatory power of characteristics than covariances for asset returns; (vii) upward sloping nominal yield curve; (viii) predictability of future bond excess returns and long term rates via the slope of the yield curve; (ix) corporate bond spread puzzle; and (x) high price of deep out-of-the-money puts. This paper provides a calibration in which those puzzles can be understood quantitatively as well. The fear of disasters can be interpreted literally, or be used to analyze how perceptions of risk affect asset prices and returns in a broad class of models.

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1 Introduction

Lately, there has been a revival of a hypothesis proposed by Rietz (1988) that the possibility of rare disasters, such as economic depressions or wars, is a major determinant of asset risk premia. Indeed, Barro (2006) has shown that, internationally, disasters have been sufficiently frequent and large to make Rietz's proposal viable and account for the high risk premium on equities.

The rare disaster hypothesis is almost always formulated with constant severity of disasters. This is useful to think about averages but cannot account for some key features of asset markets such as volatile price-dividend ratios for stocks, volatile bond risk premia, and return predictability. In this paper, I formulate a variable-severity version of the rare disasters hypothesis and investigate the impact of time-varying disaster severity on the prices of stocks and bonds as well as the predictability of their returns.[1]

I show that many asset puzzles can be qualitatively understood using this model. I then demonstrate that a parsimonious calibration allows one to understand the puzzles quantitatively provided that real and nominal variables have a sufficiently variable sensitivity to disasters (something I will argue below is plausible).

The proposed framework allows for a very tractable model of stocks and bonds in which all prices are in closed forms. In this setting, the following patterns are not puzzles but emerge naturally when the present model has just two shocks: one real for stocks and one nominal for bonds.

A. Stock market: Puzzles about the aggregates

1. Equity premium puzzle.

2. Risk-free rate puzzle.[2]

3. Excess volatility puzzle: Stock prices are more volatile than warranted by a model with a constant discount rate.

4. Aggregate return predictability: Future aggregate stock market returns are partly predicted by price/dividend and similar ratios.

   B. Stock market: Puzzles about the cross-section of stocks

5. Value/Growth puzzle: Stocks with a high (resp. low) P/D ratio have lower (resp. higher) future returns, even after controlling for their covariance with the aggregate stock market.

[1] A later companion paper, Farhi and Gabaix (2008) studies exchange rates. A brief introduction is Gabaix (2008), but almost all results appear here for the first time.

[2] For this and the above puzzle, the paper simply imports from Rietz (1988), Longstaff and Piazzesi (2004) and Barro (2006).

6. Characteristics vs. Covariances puzzle: Stock characteristics (e.g. the P/D ratio) often predict future returns as well as or better than covariances with risk factors.

C. Nominal bond puzzles

7. Yield curve slope puzzle: The yield curve slopes up on average. The premium of long-term yields over short-term yields is too high to be explained by a traditional RBC model. This is the bond version of the equity premium puzzle.

8. Long term bond return predictability: Fama-Bliss (1987), Campbell-Shiller (1991), Cochrane-Piazzesi (2005) show that a high slope of the yield curve predicts high excess returns on long term bonds and that long term interest rates will fall.

9. Credit spread puzzle: Corporate bond spreads are higher than seemingly warranted by historical default rates.

D. Options puzzles

10. Option puzzles: Deep of out-of-the-money puts have high prices. Future stock market returns can be predicted with option prices.

To understand the economics of the model, first consider bonds. Consistent with the empirical evidence (Barro and Ursua 2008), a disaster leads to a temporary jump in inflation in the model. This has a greater detrimental impact on long-term bonds, so they command a high risk premium relative to short-term bonds. This explains the upward slope of the nominal yield curve. Next, suppose that the size of the expected jump in inflation itself varies. Then the slope of the yield curve will vary and will predict excess bond returns. A high slope will mean-revert and thus predicts a fall in the long rate and high returns on long term bonds. This mechanism accounts for many stylized facts on bonds.

The same mechanism is at work for stocks. Suppose that a disaster reduces the fundamental value of a stock by a time-varying amount. This yields a time-varying risk premium which generates a time-varying price-dividend ratio and the "excess volatility" of stock prices. It also makes stock returns predictable via measures such as the dividend-price ratio. When agents perceive the severity of disasters as low, price-dividend ratios are high and future returns are low.

The model's mechanism also impacts disaster-related assets such as corporate bonds and options. If high-quality corporate bonds default mostly during disasters, then they should command a high premium that cannot be accounted for by their behavior during normal times. The model also generates option prices with a "volatility smirk," i.e. a high put price (hence implied volatility) for deep out-of-the-money puts.

After laying out the framework and solving it in closed form, I ask if some parameters can rationalize the fairly large volatility of asset prices. The average values are essentially taken from Barro and Ursua (2008)'s analysis of many countries. The calibration gives results for stocks, bonds and options consistent with empirical values. The volatilities of expectation about disaster sizes are very hard to measure directly. However, the numbers calibrated in this paper generate a steady state dispersion of anticipations that is almost certainly lower than the dispersion of realized values. By that criterion, the calibrated values in the model appear reasonable. Still, whether they are is ultimately an empirical question. It is beyond the scope and spirit of this paper to provide new historical results. This paper is essentially only theoretical – it lays out the framework of variable rare disaster, solves it, and provides a calibration.

While the model is presented as rational, it could be interpreted as behavioral. The changing beliefs about the intensity of possible disasters are very close to what the behavioral literature calls "animal spirits," and the recent rational literature calls "perception of distant risks" or "time-varying risk aversion." The model's structure gives a consistent way to think about the impact of changing "sentiment" on prices in the time-series and in the cross-section.[3]

Throughout this paper, I use the class of "linearity-generating" (LG) processes (Gabaix 2009). That class keeps all expressions in closed form. The entire paper could be rewritten with other processes (e.g. affine-yield models) albeit with considerably more complicated algebra and the need to resort to numerical solutions. The LG class and the affine class give the same expression to a first order approximation. Hence, there is little of economic consequence in the use of LG processes and their use should be viewed as an analytical convenience.

Relation to the literature A few papers address the issue of time-varying disasters. Longstaff and Piazzesi (2004) consider an economy with constant intensity of disasters, but in which stock dividends are a variable, mean-reverting share of consumption. They find a high equity premium, and highly volatile stock returns. Veronesi (2004) considers a model in which investors learn about a world economy that follows a Markov chain through two possible economic states, one of which may be a disaster state. He finds GARCH effects and apparent "overreaction." Weitzman (2007) provides a Bayesian view that the main risk is model uncertainty, as the true volatility of consumption may be much higher than the sample volatility. Unlike the present work, those papers do not consider bonds, nor study return predictability. After the present paper was circulated, Wachter (2008) proposed a quite different model with a time-varying probability of disaster, in an affine-yield framework. The present paper admits a time-varying probability of disaster, though it relies on a constant one in its calibration.

[3] In another interpretation of the model, the "disasters" are not macroeconomic disasters, but financial crises.

Within the class of rational, representative-agents frameworks that deliver time-varying risk premia, the variable rare disasters model may be a third workable framework, along with the external-habit model of Campbell-Cochrane (CC, 1999) and the long run risk model of Bansal-Yaron (BY, 2004). They have proven to be two very useful and influential models. Still, the reader might ask, why do we need another model of time-varying risk premia? The variable rare disasters framework has several useful features.

First, as emphasized by Barro (2006), the model uses the traditional iso-elastic expected utility framework like the majority of macroeconomic theory. CC and BY use more complex utility functions with external habit and Epstein-Zin (1989)-Weil (1990) utility, which are harder to embed in macroeconomic models. In ongoing work, I show how the present model (which is in an endowment economy) can be directly mapped into a production economy with traditional real-business cycle features. Hence, because it keeps the same utility function as the overwhelming majority of the rest of economics, the rare disasters idea brings us close to the long-sought unification of macroeconomics and finance (see Jermann 1998 and Boldrin, Christiano and Fisher 2001, Uhlig 2007 for attacks of this problem using habit formation). Second, the model is particularly tractable. Stock and bond prices have linear closed forms. As a result, asset prices and premia can be derived and fully understood without recourse to simulations.[4] Third, the model is based on a different hypothesis about the origins of risk premia, namely the fear of rare disasters, which has some intuitive appeal and allows agents to have moderate risk aversion. Fourth, the model easily accounts for some facts that are hard to generate in the CC and BY models. In the model, "characteristics" (such as price-dividend ratios) predict future stock returns better than market covariances, something that it is next to impossible to generate in the CC and BY models. The model also generates a low correlation between consumption growth and stock market returns, which is hard for CC and BY to achieve, as emphasized by Lustig, van Nieuwerburgh, and Verdelhan (2008).

There is a well-developed literature that studies jumps particularly with option pricing in mind. Using options, Liu, Pan and Wang (2004) calibrate models with constant risk premia and uncertainty aversion demonstrating the empirical relevance of rare events in asset pricing. Santa Clara and Yan (forth.) also use options to calibrate a model with frequent jumps. Typically, the jumps in these papers happen every few days or few months and affect consumption by moderate amounts, whereas the jumps in the rare-disasters literature happen perhaps once every 50 years, and are larger. Those authors do not study the impact of jumps on bonds and return predictability. Finally, also motivated by rare disasters, Martin (2008) develops a new technique that allows the study of models with

[4] As an example, take the Fama-Bliss and Campbell-Shiller regressions. The variable rare disaster model predicts that one should see a coefficient of 1 and -1 in those regressions, using analytical methods that allow one to precisely understand the underlying mechanism. These predictions match empirical results to a first order, and the model even quantifies higher-order deviations.

jumps. This paper's variable rare disasters model also complements the literature on long term risk – which views the risk of assets as the risk of covariance with long-run consumption (Bansal and Yaron (2004), Bansal and Shaliastovich (2008), Bekaert, Engstrom and Grenadier (2006), Gabaix and Laibson (2002), Julliard and Parker (2004)).

Section 2 presents the macroeconomic environment and the cash-flow processes for stocks and bonds. Section 3 derives equilibrium prices, and proposes a calibration. I study the model's implication for stocks and stock options in Section 4 and for bonds in Section 5. Section 6 discusses various extensions of the model. Appendix A is a gentle introduction to linearity-generating processes and Appendix B contains proofs of most of the propositions in the paper.

2 Model Setup

2.1 Macroeconomic Environment

The environment follows Rietz (1988) and Barro (2006) and adds a stochastic probability and severity of disasters. There is a representative agent with utility $E_0\left[\sum_{t=0}^{\infty} e^{-\delta t}\frac{C_t^{1-\gamma}-1}{1-\gamma}\right]$, where $\gamma \geq 0$ is the coefficient of relative risk aversion and $\delta > 0$ is the rate of time preference. She receives a consumption endowment C_t. At each period $t+1$, a disaster may happen with a probability p_t. If a disaster does not happen $C_{t+1}/C_t = e^{g_C}$ where g_C is the normal-time growth rate of the economy. If a disaster happens $C_{t+1}/C_t = e^{g_C}B_{t+1}$, where $B_{t+1} > 0$ is a random variable.[5] For instance, if $B_{t+1} = 0.8$, consumption falls by 20%. To sum up:[6]

$$\frac{C_{t+1}}{C_t} = e^{g_C} \times \begin{cases} 1 & \text{if there is no disaster at } t+1 \\ B_{t+1} & \text{if there is a disaster at } t+1 \end{cases} \tag{1}$$

The pricing kernel is the marginal utility of consumption $M_t = e^{-\delta t}C_t^{-\gamma}$, and follows:

$$\frac{M_{t+1}}{M_t} = e^{-R} \times \begin{cases} 1 & \text{if there is no disaster at } t+1 \\ B_{t+1}^{-\gamma} & \text{if there is a disaster at } t+1 \end{cases} \tag{2}$$

where $R = \delta + \gamma g_c$ is the risk-free rate in an economy that would have a zero probability of disasters. The price at t of an asset yielding a stream of dividends $(D_s)_{s\geq t}$ is: $P_t = E_t\left[\sum_{s\geq t} M_s D_s\right]/M_t$.

[5] Typically, extra i.i.d. noise is added, but given that it never materially affects asset prices it is omitted here. It could be added without difficulty. Also, a countercyclicality of risk premia could be easily added to the model without hurting its tractability.

[6] The consumption drop is permanent. One can add mean-reversion after a disaster as in Gourio (2008).

2.2 Setup for Stocks

I consider a typical stock i which is a claim on a stream of dividends $(D_{it})_{t\geq 0}$, that follows:[7]

$$\frac{D_{i,t+1}}{D_{it}} = e^{g_{iD}} \left(1+\varepsilon^D_{i.t+1}\right) \times \begin{cases} 1 & \text{if there is no disaster at } t+1 \\ F_{i,t+1} & \text{if there is a disaster at } t+1 \end{cases} \tag{3}$$

where $\varepsilon^D_{i,t+1} > -1$ is a mean zero shock that is independent of the disaster event.[8] This shock only matters for the calibration of dividend volatility. In normal times, D_{it} grows at an expected rate of g_{iD}. But, if there is a disaster, the dividend of the asset is partially wiped out following Longstaff and Piazzesi (2004) and Barro (2006): the dividend is multiplied by $F_{i,t+1} \geq 0$. $F_{i,t+1}$ is the recovery rate of the stock. When $F_{i,t+1} = 0$ the asset is completely destroyed or expropriated. When $F_{i,t+1} = 1$, there is no loss in dividend.

To model the time-variation in the asset's recovery rate, I introduce the notion of "expected resilience" H_{it} of asset i,

$$H_{it} = p_t E_t \left[B^{-\gamma}_{t+1} F_{i,t+1} - 1 \mid \text{There is a disaster at } t+1\right]. \tag{4}$$

In (4) p_t and $B^{-\gamma}_{t+1}$ are economy-wide variables while the resilience and recovery rate $F_{i,t+1}$ are stock specific though typically correlated with the rest of the economy. When the asset is expected to do well in a disaster (high $F_{i,t+1}$), H_{it} is high – investors are optimistic about the asset. In the cross-section an asset with higher resilience H_{it} is safer than one with low resilience.

To keep the notations simple from now on, I drop the "i" subscript when there is no ambiguity. I specify the dynamics of H_t directly rather than specify the individual components p_t, B_{t+1} and $F_{i,t+1}$. I split resilience H_t into a constant part H_* and a variable part $\widehat{H}_t$:

$$H_t = H_* + \widehat{H}_t$$

and postulate the following linearity-generating (LG) process for the variable part $\widehat{H}_t$:

$$\widehat{H}_{t+1} = \frac{1+H_*}{1+H_t} e^{-\phi_H} \widehat{H}_t + \varepsilon^H_{t+1} \tag{5}$$

where $E_t \varepsilon^H_{t+1} = 0$, and $\varepsilon^H_{t+1}, \varepsilon^D_{t+1}$ and the disaster event are uncorrelated variables.[9] Eq. 5 means

[7] There can be many stocks. The aggregate stock market is a priori not aggregate consumption, because the whole economy is not securitized in the stock market. Indeed, stock dividends are more volatile that aggregate consumption, and so are their prices (Lustig, van Nieuwerburgh, Verdelhan, 2007).

[8] This is, $E_t\left[\varepsilon^D_{i,t+1}\right] = E_t\left[\varepsilon^D_{i,t+1} \mid \text{Disaster at } t+1\right] = 0$.

[9] ε^H_{t+1} can be heteroskedastic – but, its variance need not be spelled out, as it does not enter into the prices.

that $\widehat{H}_t$ mean-reverts to 0 but as a "twisted" autoregressive process (Gabaix 2009 develops these twisted or "linearity-generating" processes).[10] As H_t hovers around H_*, $\frac{1+H_*}{1+H_t}$ is close to 1 and the process is an AR(1) up to second order terms: $\widehat{H}_{t+1} = e^{-\phi_H}\widehat{H}_t + \varepsilon^H_{t+1} + O\left(\widehat{H}_t^2\right)$. The Technical Appendix of Gabaix (2009) shows that the process economically behaves like an AR(1).[11] The "twist" term $\frac{1+H_*}{1+H_t}$ makes prices linear in the factors and independent of the functional form of the noise.[12] I next turn to bonds.

2.3 Setup for Bonds

The two most salient facts on nominal bonds are arguably the following. First, the nominal yield curve slopes up on average; i.e., long term rates are higher than short term rates (e.g., Campbell 2003, Table 6). Second, there are stochastic bond risk premia. The risk premium on long term bonds increases with the difference in the long term rate minus short term rate. (Campbell and Shiller 1991, Cochrane and Piazzesi 2005, Fama and Bliss 1987). These facts are considered to be puzzles, because they are not generated by the standard macroeconomic models, which generate too small risk premia (Mehra and Prescott 1985).

I propose the following explanation. When a disaster occurs, inflation increases (on average). Since very short term bills are essentially immune to inflation risk while long term bonds lose value when inflation is higher, long term bonds are riskier, so they get a higher risk premium. Hence, the yield curve slopes up. Moreover, the magnitude of the surge in inflation is time-varying which generates a time-varying bond premium. If that bond premium is mean-reverting, it generates the Fama-Bliss puzzle. Note that this explanation does not hinge on the specifics of the disaster mechanism. The advantage of the disaster framework is that it allows for formalizing and quantifying the idea in a simple way.[13]

However, the process needs to satisfy $\widehat{H}_t/(1+H_*) \geq e^{-\phi_H} - 1$, so the process is stable, and also $\widehat{H}_t \geq -p - H_*$ to ensure $F_t \geq 0$. Hence, the variance needs to vanish in a right neighborhood $\max\left(\left(e^{-\phi_H}-1\right)(1+H_*), -p-H_*\right)$. Gabaix (2007) provides more details on the stability of Linearity-Generating processes.

[10] Economically, $\widehat{H}_t$ does not jump if there is a disaster, but that could be changed with little consequence.

[11] The P/D ratio $V\left(\widehat{H}_t\right)$ follows: $V\left(\widehat{H}_t\right) = 1 + e^{-R+g}(1+H_t)E_t\left[V\left(\widehat{H}_{t+1}\right)\right]$, which is isomorphic to the example worked out in Gabaix (2009, Technical Appendix), for both twisted and non-twisted processes.

[12] Also, the "true process" is likely complicated, and both AR(1) and LG processes are approximate representations of it. There is no presumption that the true process should be exactly AR(1). Indeed, it could be that the true process is LG, and we usually approximate it with an AR(1).

[13] Several authors have models where inflation is higher in bad times, which makes the yield curve slope up (Brandt and Wang 2003, Piazzesi and Schneider 2006, Wachter 2006). This paper is part of burgeoning literature on the economic underpinning of the yield curves, see e.g. Piazzesi and Schneider (2007), and Xiong and Yan (2008). An earlier unification of several puzzles is provided by Wachter (2006), who studies a Campbell-Cochrane (1999) model, and concludes that it explains an upward sloping yield curve and the Campbell-Shiller (1991) findings. The Brandt and Wang (2003) study is also a Campbell-Cochrane (1999) model, but in which risk-aversion depends directly on inflation. In Piazzesi and Schneider (2006) inflation also rises in bad times, although in a very different model.

I now formalize the above ideas. Core inflation is i_t. The real value of money is called Q_t and evolves as:

$$\frac{Q_{t+1}}{Q_t} = \left(1 - i_t + \varepsilon_{t+1}^Q\right) \times \begin{cases} 1 & \text{if there is no disaster at } t+1 \\ F_{t+1}^{\$} & \text{if there is a disaster at } t+1 \end{cases} \tag{6}$$

where ε_{t+1}^Q has mean 0 whether or not there is a disaster at $t+1$. It is typically enough to consider the case where at all times $\varepsilon_{t+1}^Q \equiv 0$. Hence, in normal times the real value of money depreciates on average at the rate of core inflation, i_t. Following Barro (2006), there is possibility of default in disasters, where severity is indexed by the recovery rate $F_{t+1}^{\$} \geq 0$. $F_{t+1}^{\$} = 1$ means full recovery; $F_{t+1}^{\$} < 1$ means partial recovery. The default could be an outright default (e.g., for a corporate bond) or perhaps a burst of inflation that increases the price level and thus reduces the real value of the coupon. In this first pass, to isolate the bond effects, I assume the case where $H^{\$} = p_t \left(E_t \left[F_{t+1}^{\$} B_{t+1}^{-\gamma}\right] - 1\right)$ is a constant. In the calibration, I assume $p_t, F_{t+1}^{\$}$ and the distribution of B_{t+1}, to be constant. Relaxing this assumption is easy but is not central to the economics, so I do not do it here. Economically, I assume that most variations in the yield curve come from variation in inflation and inflation risk, not in the changes in the probability and severity of disasters.

I decompose inflation as $i_t = i_* + \widehat{i_t}$, where i_* is its constant part and $\widehat{i_t}$ is its variable part. The variable part of inflation follows the process:

$$\widehat{i}_{t+1} = \frac{1 - i_*}{1 - i_t} \cdot \left(e^{-\phi_i} \widehat{i}_t + 1_{\{\text{Disaster at } t+1\}} \left(j_* + \widehat{j}_t\right)\right) + \varepsilon_{t+1}^i \tag{7}$$

where ε_{t+1}^i has mean 0 and is uncorrelated with both ε_{t+1}^Q and the realization of a disaster.

This equation means first, that if there is no disaster, $E_t \widehat{i}_{t+1} = \frac{1-i_*}{1-i_t} e^{-\phi_i} i_t$, i.e. inflation follows the LG-twisted autoregressive process (Appendix A). Inflation mean-reverts at a rate ϕ_i, with the linearity–generating twist $\frac{1-i_*}{1-i_t}$ to ensure tractability. In addition, in case of a disaster, inflation jumps by an amount $j_* + \widehat{j}_t$. This jump in inflation makes long term bonds particularly risky. j_* is the baseline jump in inflation, $\widehat{j}_t$ is the mean-reverting deviation of the jump size from baseline. It follows a twisted auto-regressive process and, for simplicity, does not jump during crises:

$$\widehat{j}_{t+1} = \frac{1 - i_*}{1 - i_t} e^{-\phi_j} \widehat{j}_t + \varepsilon_{t+1}^j \tag{8}$$

where ε_{t+1}^j has mean 0. ε_{t+1}^j is uncorrelated with disasters and with ε_{t+1}^Q, but can be correlated with innovations in i_t.

Finally, Duffee (2002) and Dai and Singleton (2002) show econometric frameworks which deliver the Fama-Bliss and Campbell-Shiller results.

Two notations are useful. First, call π_t the variable part of the bond risk premium:

$$\pi_t \equiv \frac{p_t E_t \left[B_{t+1}^{-\gamma} F_{t+1}^{\$}\right]}{1+H^{\$}} \widehat{j}_t. \tag{9}$$

It is analogous to $-\widehat{H}_t$ for stocks. The second notation is only useful when the typical jump in inflation j_* is not zero and the reader is invited to skip it in the first reading. I parametrize j_* in terms of a variable $\kappa \leq \left(1-e^{-\phi_i}\right)/2$:[14]

$$\frac{p_t E_t \left[B_{t+1}^{-\gamma} F_{t+1}^{\$}\right] j_*}{1+H^{\$}} = (1-i_*)\,\kappa\left(1-e^{-\phi_i}-\kappa\right) \tag{10}$$

i.e., in the continuous time limit: $p_t E_t\left[B_{t+1}^{-\gamma} F_{t+1}^{\$}\right] j_* = \kappa\left(\phi_i - \kappa\right)$. A high κ means a high central jump in inflation if there is a disaster. For most of the paper it is enough to think that $j_* = \kappa = 0$.

2.4 Expected Returns

I conclude the presentation of the economy by stating a general Lemma about the expected returns.

Lemma 1 *(Expected returns) Consider an asset and call $P_{t+1}^{\#} = P_{t+1} + D_{t+1}$ the asset's payoff if a disaster happens at time $t+1$. Then, the expected return of the asset at t, conditional on no disasters, is:*

$$r_t^e = \frac{1}{1-p_t}\left(e^R - p_t E_t\left[B_{t+1}^{-\gamma}\frac{P_{t+1}^{\#}}{P_t}\right]\right) - 1. \tag{11}$$

In the limit of small time intervals,

$$r_t^e = R + p_t\left(1 - E_t\left[B_{t^+}^{-\gamma}\frac{P_{t^+}^{\#}}{P_t}\right]\right) = r_f + p_t E_t\left[B_{t^+}^{-\gamma}\left(1-\frac{P_{t^+}^{\#}}{P_t}\right)\right] \tag{12}$$

where t^+ denotes the values of the variables immediately after t, and

$$r_f = R - p_t E_t\left[B_{t^+}^{-\gamma} - 1\right] \tag{13}$$

is the real risk-free rate in the economy.

Equation 11 indicates that only the behavior in disasters (the $P_{t+1}^{\#}/P_t$ term) creates a risk premium. It is equal to the risk-adjusted (by $B_{t+1}^{-\gamma}$) expected capital loss of the asset if there is a disaster.

[14] Calculating bond prices in a Linearity-Generating process sometimes involves calculating the eigenvalues of its generator. I presolve by parameterizing j_* by κ. The upper bound on κ implicitly assumes that j_* is not too large.

The unconditional expected return on the asset (i.e., without conditioning on no disasters) in the continuous time limit is $r_t^e - p_t E_t \left[1 - \frac{P_{t+}^{\#}}{P_t}\right]$. Barro (2006) observes that the unconditional expected return and the expected return conditional on no disasters are very close. The possibility of disaster affects primarily the risk premium, and much less the expected loss.

3 Asset Prices and Returns

3.1 Stocks

Theorem 1 *(Stock prices) Let $h_* = \ln(1 + H_*)$ and define*

$$r_i = R - g_D - h_* \tag{14}$$

which can be called the stock's effective discount rate. The price of stock i is:

$$P_t = \frac{D_t}{1 - e^{-r_i}} \left(1 + \frac{e^{-r_i - h_*} \widehat{H}_t}{1 - e^{-r_i - \phi_H}}\right). \tag{15}$$

In the limit of short time periods, the price is:

$$P_t = \frac{D_t}{r_i} \left(1 + \frac{\widehat{H}_t}{r_i + \phi_H}\right). \tag{16}$$

The next proposition links resilience H_t and the equity premium.

Proposition 1 *(Expected stock returns) The expected returns on stock i, conditional on no disasters, are:*

$$r_{it}^e = R - H_t \tag{17}$$

where R is economy-wide but H_t is specific to stock i. The equity premium (conditional on no disasters) is $r_{it}^e - r_f = pE_t \left[B_t^{-\gamma}(1 - F_t)\right]$ where r_f is the risk-free rate derived in (13).

As expected, more resilient stocks (assets that do better in a disaster) have a lower ex ante risk premium (a higher H_t). When resilience is constant ($\widehat{H}_t \equiv 0$), Equation 16 is Barro (2006)'s expression. The price-dividend ratio is increasing in the stock's resiliency of the asset: $h_* = \ln(1 + H_*)$. In (14), R is economy-wide while g_D and h_* are stock-specific.

The key advance in Theorem 1 is that it derives the stock price with a stochastic resilience $\widehat{H}_t$. More resilient stocks (high $\widehat{H}_t$) have a higher valuation. Since resilience $\widehat{H}_t$ is volatile, price-dividend ratios are volatile, in a way that is potentially independent of innovations to dividends. Hence, the model generates a time-varying equity premium and there is "excess volatility," i.e. volatility of the stocks unrelated to cash-flow news. As the P/D ratio is stationary, it mean reverts. Thus, the model generates predictability in stock prices. Stocks with a high P/D ratio will have low returns and stocks with a low P/D ratio will have high returns. Section 4 quantifies this predictability. Proposition 10 extends equation (16) to a world that has variable expected growth rate of cash-flows in addition to variable risk premia.

3.2 Bonds

Theorem 2 *(Bond prices) In the limits of small time intervals, the nominal short term rate is* $r_t = R - H^{\$} + i_t$*, and the price of a nominal zero-coupon bond of maturity* T *is:*

$$Z_t(T) = e^{-\left(R-H^{\$}+i_{**}\right)T}\left(1 - \frac{1-e^{-\psi_i T}}{\psi_i}\left(i_t - i_{**}\right) - \frac{\frac{1-e^{-\psi_i T}}{\psi_i} - \frac{1-e^{-\psi_\pi T}}{\psi_\pi}}{\psi_\pi - \psi_i}\pi_t\right), \tag{18}$$

where i_t *is inflation,* π_t *is the bond risk premium,* $i_{**} \equiv i_* + \kappa$*,* $\psi_i \equiv \phi_i - 2\kappa$*,* $\psi_\pi \equiv \phi_j - \kappa$*, and* κ *parametrizes the permanent risk of a jump in inflation (10). The discrete-time expression is in (52) of Appendix B.*

Theorem 2 gives a closed-form expression for bond prices. As expected, bond prices decrease with inflation and with the bond risk premium. Indeed, expressions $\frac{1-e^{-\psi_i T}}{\psi_i}$ and $\frac{\frac{1-e^{-\psi_i T}}{\psi_i} - \frac{1-e^{-\psi_\pi T}}{\psi_\pi}}{\psi_\pi - \psi_i}$ are non-negative and increasing in T. The term $\frac{1-e^{-\psi_i T}}{\psi_i} i_t$ simply expresses that inflation depresses nominal bond prices and mean-reverts at a (risk-neutral) rate ψ_i. The bond risk premium π_t affects all bonds but not the short-term rate.

When $\kappa > 0$ (resp. $\kappa < 0$) inflation typically increases (resp. decreases) during disasters. While ϕ_i (resp. ϕ_j) is the speed of mean-reversion of inflation (resp. of the bond risk premium, which is proportional to j_t) under the physical probability, ψ_i (resp. ψ_π) is the speed of mean-reversion of inflation (resp. of the bond risk premium) under the risk-neutral probability.

I next calculate expected bond returns, bond forward rates, and yields.

Proposition 2 *(Expected bond returns) Conditional on no disasters, the short-term real return on a short-term bill is:* $R^e(0) = R - H^{\$}$ *and the real excess return on the bond of maturity* T *is:*

$$\begin{aligned} R^e(T) - R^e(0) &= \frac{\frac{1-e^{-\psi_i T}}{\psi_i}\left(\kappa\left(\psi_i + \kappa\right) + \pi_t\right)}{1 - \frac{1-e^{-\psi_i T}}{\psi_i}\left(i_t - i_{**}\right) + \frac{\frac{1-e^{-\psi_i T}}{\psi_i} - \frac{1-e^{-\psi_\pi T}}{\psi_i}}{\psi_\pi - \psi_i}\pi_t} & (19) \\ &= T\left(\kappa\left(\psi_i + \kappa\right) + \pi_t\right) + O\left(T^2\right) + O\left(\pi_t, i_t, \kappa\right)^2 & (20) \\ &= T p_t E_t\left[B_{t^+}^{-\gamma} F_{t^+}^{\$}\right]\left(j_* + \widehat{j}_t\right) + O\left(T^2\right) + O\left(\pi_t, i_t, \kappa\right)^2. & (21) \end{aligned}$$

Expression (21) shows the first order value of the bond risk premium for bonds of maturity T. It is the maturity T of the bond times an inflation premium, $p_t E_t \left[B_{t^+}^{-\gamma} F_{t^+}^{\$} \right] \left(j_* + \widehat{j}_t \right)$. The inflation premium is equal to the risk-neutral probability of disasters (adjusting for the recovery rate), $p_t E_t \left[B_{t^+}^{-\gamma} F_{t^+}^{\$} \right]$, times the expected jump in inflation if there is a disaster, $j_* + \widehat{j}_t$. We note that a lower recovery rate shrinks risk premia, a general feature we will explore in more detail in Section 5.5.

Lemma 2 *(Bond yields and forward rates) The forward rate, $f_t(T) \equiv -\partial \ln Z_t(T) / \partial T$ is:*

$$f_t(T) = R - H^{\$} + i_{**} + \frac{e^{-\psi_i T}(i_t - i_{**}) + \frac{e^{-\psi_i T} - e^{-\psi_\pi T}}{\psi_\pi - \psi_i} \pi_t}{1 - \frac{1-e^{-\psi_i T}}{\psi_i}(i_t - i_{**}) - \frac{\frac{1-e^{-\psi_i T}}{\psi_i} - \frac{1-e^{-\psi_\pi T}}{\psi_i}}{\psi_\pi - \psi_i} \pi_t} \tag{22}$$

and admits the Taylor expansion:

$$\begin{aligned} f_t(T) &= R - H^{\$} + i_{**} + e^{-\psi_i T}(i_t - i_{**}) + \frac{e^{-\psi_i T} - e^{-\psi_\pi T}}{\psi_\pi - \psi_i} \pi_t + O(i_t - i_{**}, \pi_t)^2 & (23) \\ &= R - H^{\$} + i_{**} + \left(1 - \psi_i T + \frac{\psi_i^2 T^2}{2}\right)(i_t - i_{**}) + \left(T - \frac{\psi_i + \psi_\pi}{2} T^2\right) \pi_t & (24) \\ &\quad + O\left(T^3\right) + O(i_t - i_{**}, \pi_t)^2 . \end{aligned}$$

The bond yield is $y_t(T) = -(\ln Z_t(T))/T$ with $Z_t(T)$ given by (18), and its Taylor expansion is given in Eq. 53-55.

The forward rate increases with inflation and the bond risk premia. The coefficient of inflation decays with the speed of mean-reversion of inflation, ψ_i, in the "risk-neutral" probability. The coefficient of the bond premium, π_t, is $\frac{e^{-\psi_i T} - e^{-\psi_\pi T}}{\psi_\pi - \psi_i}$, hence has value 0 at both very short and very long maturities and is positive hump-shaped in between. The reason is that very short term bills, being safe, do not command a risk premium, and long term forward rates also are essentially constant (Dybvig, Ingersoll and Ross 1996). Thus, the time-varying risk premium only affects intermediate maturities of forwards.

3.3 A Calibration

I propose the following calibration of the model's parameters, expressed in annualized units. I assume that time-variation disaster risk enters through the recovery rate F for stocks and through the potential jump in inflation j for bonds.

Macroeconomy. In normal times, consumption grows at rate $g_c = 2.5\%$. To keep things parsimonious, the probability and conditional severity of macroeconomic disasters are constant. The disaster probability is $p = 3.63\%$, Barro and Ursua (2008)'s estimate. I take $\gamma = 4$, for which Barro and Ursua calculate $E\left[B^{-\gamma}\right] = 5.29$ so that the utility-weighted mean recovery rate of consumption is $E\left[B^{-\gamma}\right]^{-1/\gamma} = \overline{B} = 0.66$. Because of risk aversion, bad events get a high weight: the modal loss is less severe.[15] The key number is the risk-neutral probability of disasters, $pE\left[B^{-\gamma}\right] = 0.192$. This high risk-neutral probability allows the model to calibrate a host of high risk premia. Following Barro and Ursua, I set the rate of time preference to match a risk free rate of 1%, so, in virtue of Eq. 13, the rate of time preference is $\delta = 6.6\%$.

Stocks. I take a growth rate of dividends $g_D = g_C$, consistent with the international evidence (Campbell 2003, Table 3). The volatility of the dividend is $\sigma_D = 11\%$, as in Campbell and Cochrane (1999).

The speed of mean-reversion of resilience ϕ_H, is the speed of mean-reversion of the price/dividend ratio. It has been carefully examined in two recent studies based on US data. Lettau and van Nieuwerburgh (2008) find $\phi_H = 9.4\%$. However, they find $\phi_H = 26\%$ when allowing for a structural break in the time series, which they propose is warranted. Cochrane (1988) finds $\phi_H = 6.1\%$, with a s.e. of 4.7%. I take the mean of those three estimates, which leads to $\phi_H = 13\%$. Given these ingredients, Appendix A specifies a volatility process for H_t.

To specify the volatility of the recovery rate F_t, I specify that it has a baseline value $F_* = \overline{B}$, and support $F_t \in [F_{\min}, F_{\max}] = [0, 1]$. That is, if there is a disaster dividends can do anything between losing all their value and losing no value. The process for H_t then implies that the corresponding average volatility for F_t, the expected recovery rate of stocks in a disaster, is 10%. This may be considered to be a high volatility. Economically, it reflects the fact that it seems easy for stock market investors to alternatively feel extreme pessimism and optimism (e.g., during the large turning points around 1980, around 2001 and around 2008). In any case, this perception of the risk for F_t is not observable directly, so the calibration does not appear to contradict any known fact about observable quantities.

Bonds. For simplicity and parsimony, I consider the case when inflation does not burst during disasters, $F^{\$} = 1$. Bonds and inflation data come from CRSP. Bond data are monthly prices of zero-coupon bonds with maturities of 1 to 5 years, from June 1952 through September 2007. In the same time sample, I estimate the inflation process as follows. First, I linearize the LG process for inflation, which becomes: $i_{t+1} - i_* = e^{-\phi_i \Delta t}\left(i_t - i_*\right) + \varepsilon_{t+1}^i$. Next, it is well-known that inflation, observed

[15] There is an active literature centering around the basic disaster parameters: see Barro and Ursua (2008), Barro, Nakamura, Steinsson and Ursua (2008) who find estimates consistent with the initial Barro (2006) numbers; Ghosh and Julliard (2008) find a lower importance of disaster. The methodological debate, which involves missing observations, for instance due to closed stock markets, price controls, the measurement of consumption, and the very definition of disasters, is likely to continue for years to come. However, it is still legitimate to think about the consequences of disasters, even if the calibrating parameters are not yet entirely stabilized. In additional, the model has a great deal of flexibility. By doubling γ, the value of $E\left[B^{-\gamma}\right]$ is more than squared, so that we can reach values of $E\left[B^{-\gamma}\right] = 25$. This flexibility is not needed in this calibration.

at the monthly frequency, contains a substantial amount of high-frequency, transitory component, which in part is due to measurement error. The model accommodates this. Call $I_t = i_t - \varepsilon_t^Q$ the measured inflation, while i_t is the core inflation. I estimate inflation as a Kalman filter, with $i_{t+1} = C_1 + C_2 i_t + \varepsilon_{t+1}^i$ for the core inflation, and $I_t = i_t - \varepsilon_t^Q$ for the noisy measurement of inflation. Estimation is at the quarterly frequency, and yields $C_2 = 0.954$ (s.e. 0.020), i.e. mean-reversion of inflation is $\phi_i = 0.18$ in annualized values. Also, the annualized volatility of innovations in core inflation is $\sigma_i = 1.5\%$. I have also checked that estimating the process for i_t on the nominal short rate (as recently done by Fama 2006) yields substantially the same conclusion. Finally, I set i_* at the mean inflation, 3.7%. (The small non-linearity in the LG term process makes i_* differ from the mean of i_t by only a negligible amount).

To assess the process for j, I consider the 5-year slope, $s_t = y_t(5) - y_t(1)$. Eq. 55 shows that, conditional on no disasters, it follows (up to second order terms), $s_{t+1} = a + e^{-\phi_j \Delta t} s_t + b i_t + \varepsilon_{t+1}^s$, where Δt is the frequency of "a period" (e.g., a quarter means $\Delta t = 1/4$). I estimate this process at a quarterly frequency. The coefficient on s_t is 0.795 (s.e. 0.043). This yields $\phi_j = 0.92$. The standard deviation of innovations to the slope is 0.92%.

The mean value of the 5-year slope is 0.57% in the data. Matching it in the model leads to a spread of very long term yields to short term yields of $\kappa = 2.6\%$.[16] By (10), this implies an inflation jump during disasters of $j_* = 2.1\%$. As a comparison, Barro and Ursua (2008) find a median increase of inflation during disasters of 2.4%.[17] This is heartening, but one must keep in mind that Barro and Ursua find that the average increase in inflation during disasters is equal to 109% – because of hyperinflations, inflation is very skewed. I conclude that a jump in inflation of 2.1% is consistent with the historical experience. As there is considerable variation in the actual jump in inflation, there is much room for variations in the perceived jump in inflation, $j_t = j_* + \widehat{j}_t$ – something that the calibration indeed will deliver.

We saw that empirically, the standard deviation of the innovations to the 5-year spread is 0.92% (in annualized values), while in the model it is: $(K_5 - K_1)\,\sigma_\pi$, with $K_T \equiv \frac{\frac{1-e^{-\psi_i T}}{\psi_i} - \frac{1-e^{-\psi_\pi T}}{\psi_i}}{(\psi_\pi - \psi_i) T}$. Hence we calibrate $\sigma_\pi = 2.9\%$. As a result, the standard deviation of the 5-year spread is $(K_5 - K_1)\,\sigma_\pi / \sqrt{2\phi_j} = 0.68\%$, while in the data it is 0.79%. Hence, the model is reasonable in terms of observables.

An important non-observable is the perceived jump of inflation during a disaster, j. Its volatility is $\sigma_j = \sigma_\pi / \left(pE\left[B^{-\gamma}\right]\right) = 15.4\%$, and its population standard deviation is $\sigma_j / \sqrt{2\phi_j} = 11\%$. This is

[16] I consider the base line value of the yield, which from (18) is $y_t(T) = r_f^{\$} + \kappa + \ln\left(1 - \frac{1-e^{-\psi_i T}}{\psi_i}\kappa\right)/T$, with $\psi_i = \phi_i - 2\kappa$, and I compute that value of κ that ensures $y_t(5) - y_t(1) = 0.0057$. The fact that $y_t(T) = r_f^{\$} + \kappa(\phi_i - \kappa) T/2 + o(T)$ also offers an approximate, transparent κ.

[17] They find a median inflation rate of 6.6% during disasters, compared to 4.2% for long samples taken together.

arguably high – though it does not violate the constraint that the actual jump in inflation should be more dispersed than its expectation. One explanation is that the yield spread has some high-frequency transitory variation that leads to a very high measurement of ϕ_j; with a lower value one would obtain a considerably lower value of σ_j, around 4.8%. Another interpretation is that the demand for bonds shifts at a high frequency (perhaps for liquidity reasons). While this is captured by the model as a change in perceived inflation risk, it could be linked to other factors. In any case, we shall see that the model does well in a series of dimensions explored in Section 5.

I next turn to the return predictability generated by the model. Sometimes, I use simulations, the detailed algorithm for which is in the Online Appendix.

4 Stocks: Predictability and Options

4.1 Average Levels

The equity premium (conditional on no disasters) is $R^e - r_f = p\left(E\left[B^{-\gamma}\right](1-F_*)\right) = 6.5\%$. The unconditional equity premium is 5.3% (the above value, minus $p\left(1-\overline{B}\right)$). The difference between those two premia is 0.8%. So, as in Barro (2006), the excess returns of stocks mostly reflect a risk premium not a peso problem.[18] The mean value of the price/dividend ratio is 18.2 (and is close to Eq. 16, evaluated at $\widehat{H}_t = 0$), in line with the empirical evidence reported in Table 1. The central value of the D/P ratio is $r_i = 5.0\%$.

4.2 Aggregate Stock Market Returns: Excess Volatility and Predictability

The model generates "excess volatility" and return predictability.

"Excess" Volatility Consider (16), $P_t/D_t = \left(1 + \frac{\widehat{H}_t}{r_i+\phi_H}\right)/r_i$. As stock market resilience $\widehat{H}_t$ is volatile so are stock market prices and P/D ratios. Table 1 reports the numbers. The standard deviation of $\ln(P/D)$ is 0.27. Volatile resilience yields a volatility of the log of the price / dividend ratio equal to 10%. The volatility of equity returns is 15%.[19] I conclude that the model can quantitatively account for an "excess" volatility of stocks through a stochastic risk-adjusted severity of disasters.[20]

[18] Note that this explanation for the equity premium is very different from the one proposed in Brown, Goetzmann and Ross (1995), which centers around survivorship bias.

[19] For parsimony, innovations to dividends and resilience are uncorrelated.

[20] Also, in a sample with rare disasters, changes in the P/D ratio mean only change in future returns, not future dividends. This is in line with the empirical findings of Campbell and Cochrane (1999, Table 6).

Table 1: Some Stock Market Moments.

	Data	Model
Mean P/D	23	18.2
Stdev $\ln P/D$	0.33	0.30
Stdev of stock returns	0.18	0.15

Explanation: Stock market moments. The data are Campbell (2003, Table 1 and 10)'s calculation for the USA 1891–1997.

Predictability Consider (16) and (17). When $\widehat{H}_t$ is high, (17) implies that the risk premium is low and P/D ratios (16) are high. Hence, the model generates that when the market-wide P/D ratio is lower than normal, subsequent stock market returns will be higher than usual. This is the view held by a number of financial economists (e.g. Campbell and Schiller 1988, Cochrane 2008) although it is not undisputed (Goyal and Welch 2008). The model predicts the following magnitudes for regression coefficients.

Proposition 3 *(Coefficient in a predictive regression of stock returns via P/D ratios) Consider the predictive regression of the return from holding the stock from t to $t+T$, $r_{t\to t+T}$ on the initial log price-dividend ratio, $\ln (D/P)_t$:*

$$\text{Predictive regression: } r_{t\to t+T} = \alpha_T + \beta_T \ln (D/P)_t + \text{noise}. \tag{25}$$

In the model for small holding horizons T the slope is to the leading order: $\beta_T = (r_i + \phi_H)\, T$, where r_i is the stock's effective discount rate (14) and ϕ_H is the speed of mean-reversion of resilience, hence of the P/D ratio. For the regression:

$$\text{Predictive regression: } r_{t\to t+T} = \alpha_T + \beta'_T (D/P)_t + \text{noise} \tag{26}$$

the coefficient is, to the leading order: $\beta'_T = (1 + \phi_H / r_i)\, T$.

This intuition for the value of β_T is thus. First, the slope is proportional to T simply because returns over a horizon T are proportional to T. Second, when the P/D ratio is lower than baseline by 1%, it increases returns through two channels: the dividend yield is higher by r_i% and mean-reversion of the price-dividend ratio creates capital gains of ϕ%.

Using the paper's calibration of $r_i = 5\%$ and $\phi_H = 13\%$, Proposition 3 predicts a slope coefficient $\beta_1 = 0.18$ at a one-year horizon. This prediction is in line with the careful estimates of Lettau and van Nieuwerburgh (2008) who find a β_1 value of 0.23 in their preferred specification. Also,

Table 2: Predicting Returns with the Dividend-Price Ratio

	Data			Model	
Horizon	Slope	s.e.	R^2	Slope	R^2
1	0.11	(0.053)	0.04	0.17	0.06
4	0.42	(0.18)	0.12	0.45	0.19
8	0.85	(0.20)	0.29	0.79	0.30

Explanation: Predictive regression $E_t\left[r_{t\to t+T}\right] = \alpha_T + \beta_T \ln\left(D/P\right)_t$, at horizon T (annual frequency). The data are Campbell (2003, Table 10 and 11B)'s calculation for the US 1891–1997. The model's predictions are derived analytically in Proposition 3.

Cochrane (2008) runs regression (26) at annual horizon and finds $\beta_1' = 3.8$ with a standard error of 1.6. Proposition 3 predicts $\beta_1^{'} = 3.6$.

I conclude that the model is successful not only at matching the level, but also the variation and predictability of the stock market. This study serves as a warm up for the more advanced understanding of options and nominal bonds, to which we proceed next.

4.3 Options, Tail risk, and Equity Premium

This section provides a simple way to derive a closed-form value for options, which offer a potential way to measure disasters. The price of a European one-period put on a stock with strike K expressed as a ratio to the initial price is: $V_t = E\left[\frac{M_{t+1}}{M_t}\left(K - \frac{P_{t+1}}{P_t}\right)^+\right]$, where $x^+ \equiv \max(x, 0)$.

Economically, I assume that in a disaster most of the option value comes from the disaster, not from "normal times" volatility.[21] In normal times returns are log-normal. However, if there is a disaster, stochasticity comes entirely from the disaster (there is no Gaussian u_{t+1} noise). Under these assumptions, the option price has a nice form. Recall that Theorem 1 yielded $P_t/D_t = a + b\widehat{H}_t$ for two constants a and b. Hence, $E_t\left[\frac{P_{t+1}}{P_t} \mid \text{No disaster at } t+1\right] = e^{\mu}$, with $e^{\mu} \equiv \frac{a + b\frac{e^{-\phi_H}\widehat{H}_t}{1+e^{-h^*}\widehat{H}_t}}{a + b\widehat{H}_t} e^{g_D}$. I therefore parametrize the stochasticity according to:

$$\frac{P_{t+1}}{P_t} = e^{\mu} \times \begin{cases} e^{\sigma u_{t+1} - \sigma^2/2} & \text{if there is no disaster at } t+1 \\ F_{t+1} & \text{if there is a disaster at } t+1 \end{cases} \tag{27}$$

where u_{t+1} is a standard Gaussian variable. The above structure takes advantage of the flexibility in the modelling of the noise in $\widehat{H}_t$ and D_t. Rather than modelling them separately, I assume that

[21] The advantage of this discrete-time formulation is that, in the period, only one disaster happens. Hence, one avoids the infinite sums of Naik and Lee (1990), which lead to infinite sums of the probability of 0, 1, 2... disasters.

their aggregate gives exactly a log normal noise.[22] At the same time, (27) is consistent with the processes and prices in the rest of the paper.

Proposition 4 *(Put price) The value of a put with strike K (the fraction of the initial price at which the put is in the money) with maturity next period is $V_t = V_t^{ND} + V_t^D$ with V_t^{ND} and V_t^D corresponding to the events with no disasters and with disasters respectively:*

$$V_t^{ND} = e^{-R+\mu}(1-p_t)\, V_{Put}^{BS}\left(Ke^{-\mu}, \sigma\right) \quad (28)$$

$$V_t^{D} = e^{-R+\mu} p_t E_t \left[B_{t+1}^{-\gamma}\left(Ke^{-\mu} - F_{t+1}\right)^{+}\right] \quad (29)$$

where $V_{Put}^{BS}(K, \sigma)$ is the Black-Scholes value of a put with strike K, volatility σ, initial price 1, maturity 1, and interest rate 0.

Proposition 4 suggests a way to extract key structural parameters of disasters from options data. Stocks with a higher put price (controlling for "normal times" volatility) should have a higher risk premium, because they have higher future expected returns. Evaluating this prediction would be most interesting. Supportive evidence comes from Bollerslev, Tauchen and Zhou (forth.) and Drechsler and Yaron (2008). They find that when put prices (sometimes proxied by the VIX index) are high, subsequent stock market returns are high. This is exactly what a disaster based model predicts.

I now ask whether the model's earlier calibration yields good values for options. I calculate the model's Black-Scholes implied volatility of puts with a 1 month maturity. I am very grateful to Stephen Figlewski for providing the empirical implied volatility of 1-month options on the S&P 500, from January 2001 to February 2006, obtained with the interpolation method described in Figlewski (2008). With different methods, Du (2008) and Shasliastovich (2008) also find similar empirical implied volatilities.

Figure 1 reports the implied volatility, from the data, as well as the in the calibration. The correspondence is quite good, despite the fact that no extra parameter was tuned to match options prices. Hence, I conclude that in a first pass and for the maturity presented here, the variable rare disaster model gets options prices correct.[23] Of course, a more systematic study would be desirable. In ongoing work, Farhi, Fraiberger, Gabaix, Ranciere and Verdelhan (2009) investigate the link between currency option prices and currency levels, finding good support for the existence of a disaster risk premium.

[22] The Online Appendix provides a general way to ensure that this is possible.
[23] This conclusion is consistent with Du (2008), who calibrates a model with rare disasters and habit formation.

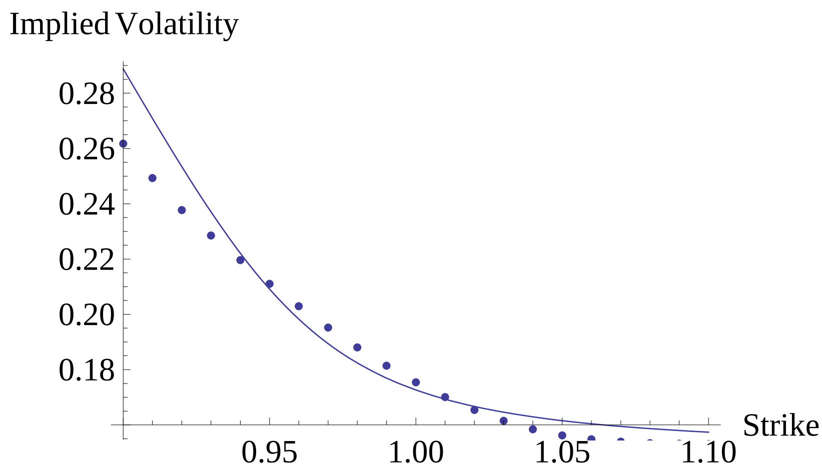

Figure 1: This Figure shows the Black-Scholes annualized implied volatility of a 1-month put on the stock market. The solid line is from the model's calibration. The dots are the empirical average (January 2001 - February 2006) for the options on the S&P 500 index, calculated as in Figlewski (2008). The initial value of the market is normalized to 1. The implied volatility on deep out-of-the-money puts is higher than the implied volatility on at-the-money puts, which reflects the probability of rare disasters.

5 Bond Premia and Yield Curve Puzzles

In this section, I show how the model matches key facts on bond return predictability.

5.1 Excess Returns and Time-Varying Risk Premia

Bonds carry a time-varying risk premium Eq. 20 indicates that bond premia are (to a first order) proportional to bond maturity T. This is the finding of Cochrane and Piazzesi (2005). The one factor here is the inflation premium π_t which is compensation for a jump in inflation if a disaster happens. The model delivers this because a bond's loading of inflation risk is proportional to its maturity T.

The nominal yield curve slopes up on average Suppose that when the disaster happens, inflation jumps by $j_* > 0$. This leads to a positive parametrization κ of the bond premia (Eq. 10). The typical nominal short term rate (i.e., the one corresponding to $i_t = i_*$) is $r = R - H^{\$} + i_*$ while the long term rate is $r + \kappa$ (i.e., $-\lim_{T\to\infty} \ln Z_t(T)/T$). Hence, the long term rate is above the short term rate by $\kappa > 0$.[24] The yield curve slopes up. Economically this is because long maturity

[24] If inflation fell during disasters, then we would have $\kappa < 0$, and the average nominal yield curve would slope down.

bonds are riskier, so they command a risk premium.

5.2 The Forward Spread Predicts Bond Excess Returns (Fama-Bliss)

Fama Bliss (1987) regress short-term excess bond returns on the forward spread, i.e. the forward rate minus the short-term rate:

$$\text{Fama-Bliss regression: Excess return on bond of maturity } T = \alpha_T + \beta_T \cdot (f_t(T) - r_t) + \text{noise}. \tag{30}$$

The expectation hypothesis yields constant bond premia, hence predicts $\beta_T = 0$. I next derive the model's prediction.

Proposition 5 *(Coefficient in the Fama-Bliss regression) The slope coefficient* β_T *of the Fama-Bliss regression (30) is given in (56) of Appendix B. When* $var(\pi_t) \gg var(i_t)\psi_i^2$ *(i.e., changes in the slope of the forward curve come from changes in the bond risk premium rather than changes in the drift of the short term rate),*

$$\beta_T = 1 + \frac{\psi_i}{2} T + O\left(T^2\right). \tag{31}$$

When $var(\pi_t) = 0$ *(no risk premium shocks) the expectation hypothesis holds and* $\beta_T = 0$*. In all cases, the slope* β_T *is positive and eventually goes to 0,* $\lim_{T\to\infty} \beta_T = 0$.

To understand the economics of the previous proposition, consider the variable part of the two sides of the Fama-Bliss regression (30). The excess return on a $T-$maturity bond is approximately $T\pi_t$ (see Eq. 20) while the forward spread is $f_t(T) - r_t \simeq T\pi_t$ (see Eq. 24). Both sides are proportional to $\pi_t T$. Thus, the Fama-Bliss regression (30) has a slope equal to 1 which is the leading term of (31).

This value $\beta_T = 1$ is precisely what Fama and Bliss have found, a finding confirmed by Cochrane and Piazzesi (2005). This is quite heartening for the model.[25] Table 3 reports the results. We see that, to a good approximation the coefficient is close to 1. We also see that as maturity increases, coefficients initially rise but then fall at long horizons, as predicted by Proposition 5. Economically, the slope of $\beta_T = 1$ means that most of the variations in the slope of the yield curve are due to variations in risk-premium, not to the expected change of inflation.

[25] Other models, if they have a time-varying bond risk premium proportional to the maturity of the bond, would have a similar success, if they have the same LG moments as this model (see section 6.1). So, the model illustrates a generic mechanism that explains the Fama-Bliss result.

Table 3: Fama-Bliss Excess Return Regression

	Data			Model	
Maturity T	β	(s.e.)	R^2	β	R^2
2	0.99	(0.33)	0.16	1.33	0.34
3	1.35	(0.41)	0.17	1.71	0.23
4	1.61	(0.48)	0.18	1.84	0.14
5	1.27	(0.64)	0.09	1.69	0.08

Explanation: The regressions are the excess returns on a zero-coupon bond of maturity T, regressed on the spread between the T forward rate and the short term rate: $rx_{t+1}(T) = \alpha + \beta(f_t(T) - f_t(1)) + \varepsilon_{t+1}(T)$. The unit of time is one year. The empirical results are from Cochrane and Piazzesi (2005, Table 2). The expectation hypothesis implies $\beta = 0$. The model's predictions are derived analytically in Proposition 5.

5.3 The Slope of the Yield Curve Predicts Future Movements in Long Rates (Campbell Shiller)

Campbell and Shiller (CS, 1991) find that a high slope of the yield curve predicts that future long term rates will fall. CS regress changes in yields on the spread between the yield and the short-term rate:

$$\text{Campbell-Shiller regression: } \frac{y_{t+\Delta t}(T-\Delta t) - y_t(T)}{\Delta t} = a + \beta_T \cdot \frac{y_t(T) - y_t(0)}{T} + \text{noise} \tag{32}$$

The expectation hypothesis predicts $\beta_T = 1$. However, CS find negative β_T's, with a roughly affine shape as a function of maturity (see Table 4). This empirical result is predicted by the model, as the next Proposition shows.

Proposition 6 *(Coefficient in the Campbell-Shiller regression) When $\phi_i^2 var(i_t)/var(\pi_t) \ll 1$, $\kappa \ll 1$, the value of the slope coefficient in the Campbell-Shiller (1991) regression (32) is:*

$$\beta_T = -\left(1 + \frac{2\psi_\pi - \psi_i}{3} T\right) + o(T) \;\; when\ T \to 0 \tag{33}$$

$$= -\psi_\pi T + o(T) \;\; when\ T \gg 1 \tag{34}$$

Eq. 57 gives the exact expression for β_T.

Table 4 also contains simulation results of the model's predictions. They are in line with CS's results. To understand the economics better, I use a Taylor expansion in the case where inflation

Table 4: Campbell-Shiller Yield Change Regression

	Data		Model
Maturity T	β	(s.e.)	β
3	-0.15	(0.28)	-1.03
6	-0.83	(0.44)	-1.16
12	-1.43	(0.60)	-1.41
24	-1.45	(1.00)	-1.92
48	-2.27	(1.46)	-2.83

Explanation: The regressions are the change in bond yield on the slope of the yield curve: $y_{t+1}(T-1) - y_t(T) = \alpha + \frac{\beta}{T-1}(y_t(T) - y_t(1)) + \varepsilon_{t+1}(T)$ The time unit is one month. The empirical results are from Campbell, Lo, MacKinlay (1997, Table 10.3). The expectation hypothesis implies $\beta = 1$. The model's predictions are derived analytically in Proposition 6.

is minimal. The slope of the yield curve is, to the leading order:

$$\text{Slope of the yield curve} \equiv \frac{y_t(T) - y_t(0)}{T} = \frac{\pi_t}{2} + O(T).$$

Hence, to a first order approximation (when inflation changes are not very predictable) the slope of the yield curve reflects the bond risk premium. The change in yield is (the proof of Proposition 6 justifies this)

$$\frac{y_{t+\Delta t}(T - \Delta t) - y_t(T)}{\Delta t} \simeq -\frac{\partial y_t(T)}{\partial T} = \frac{-\pi_t}{2} + O(T).$$

Hence, the CS regression yields a coefficient of -1, to the leading order. Economically, it means that a high bond premium increases the slope of the yield curve (by $\pi_t/2$).

As bond maturity increases, Proposition 6 predicts that the coefficient in the CS regression becomes more and more negative. The economic reason is the following. For long maturities, yields have vanishing sensitivity to the risk premium (as in Dybvig, Ingersoll and Ross 1996) which the model says has the shape $y_t(T) = a + b\pi_t/T + o(1/T)$ for some constants a, b. So the slope of the yield curve varies as $b\pi_t/T^2$. But the expected change in the yield is $-b\phi_\pi \pi_t/T$. So the slope in the CS regression (32) is $\beta_T \sim -\phi_\pi T$.[26]

In Table 4 we see that the fit between theory and evidence is quite good. The only poor fit is at small maturity. The CS coefficient is closer to 0 than in the model. The short term rate has a larger predicable component at short term horizons than in the model. For instance, this could reflect a short-term forecastability in Fed Funds rate changes. That feature could be added to the model as

[26] The expression for β_T shows that when the predictability due to inflation is non-negligible, the CS coefficient should go to 1 for very large maturities.

in Section 6.4.2. Given the small errors in fit, it is arguably better not to change the baseline model which broadly accounts for the CS finding. Economically, the CS finding reflects the existence of a stochastic one-factor bond risk premium.

5.4 Explaining Cochrane and Piazzesi (2005)

Cochrane and Piazzesi (CP, 2005) establish that (i) a parsimonious description of bond premia is given by a stochastic one-factor risk premium, (ii) (zero-coupon) bond premia are proportional to bond maturity, and (iii) this risk premium is well proxied empirically by a "tent-shape" linear combination of forward rates. Eq. 20 delivers their first two findings: there is a single bond risk factor π_t, and the loading on it is proportional to bond maturity. Economically, it is because a bond of maturity T has a sensitivity to inflation risk approximately proportional to T.

To understand CP's third finding, rewrite (23) as:

$$f_t(T) = F(T) + e^{-\psi_i T} i_t + G(T)\pi_t, \qquad G(T) \equiv \frac{e^{-\psi_i T} - e^{-\psi_\pi T}}{\psi_\pi - \psi_i}. \tag{35}$$

The economic intuition for the CP "tent shape" effect to predict π_t is as follows. The forward rate of maturity T has a loading $G(T)$ on the bond risk premium π_t. $G(T)$ has a tent-shape: $G(0) = G(\infty) = 0$, and $G(T) > 0$ for $T > 0$. We saw earlier (after Lemma 2) that the economic reason for this tent-shape of $G(T)$ is that short term bonds have no inflation risk premium, and long term forwards are constant (in this model, $f_t(\infty) = R - H^{\$} + i_{**}$). So, to capture the bond risk premium, a tent-shape $\sum_{T=1}^{5} w_T f_t(T)$ combination for forwards predicts the bond risk premium. The simple $(\sum_{T=1}^{5} w_T)$ and maturity-weighted $(\sum_{T=1}^{5} T w_T)$ sum of the weights should be roughly 0, so as to eliminate $e^{-\psi_i T} i_t$ up to second order terms.

This reasoning leads me to ask: is there a simple combination of forward rates which one might expect to robustly proxy for the risk premia? The next Proposition advances novel "estimation-free" combinations of forwards and yields:[27]

Proposition 7 *(Estimation-free combinations of forwards and yields to proxy the bond risk premium) Consider some maturities a and b, and the following "estimation-free" combinations of forwards and yields:*

$$\begin{aligned} CP'_t &\equiv \left[-f_t(a) + 2f_t(a+b) - f_t(a+2b)\right]/b^2 \\ CP''_t &\equiv \left[-2y_t(2a) + 3y_t(3a) - y_t(5a)\right]/a^2 \end{aligned}$$

[27] Lettau and Wachter (2007) proposed earlier another combination of theoretical factors to obtain the risk premium, but it requires parameter estimation, whereas CP' and CP'' do not.

where $f_t(T)$ and $y_t(T)$ are respectively forwards and yields of maturity T. Then, up to third order terms, for small a and b, $CP'_t = CP''_t = (\psi_i + \psi_j)\pi_t$, hence they are proportional to the bond risk premium.

For instance, with $a = 1$, $b = 2$, the estimation-free factors are

$$CP' = (-f(1) + 2f(3) - f(5))/4, \qquad CP'' = -2y(2) + 3y(3) - y(5) \tag{36}$$

Proposition 7 suggests that CP' and CP'' could be used in practice to proxy for the bond risk premia, without requiring a preliminary estimation. The main virtue of CP'' is that it uses few differences in yields and, hence, might be relatively robust to measurement errors.

Simulations show that CP' and CP'' predict the bond premium with an accuracy close to that of an unconstrained combination of the five forwards $f(1), ..., f(5)$: the constrained R^2's are 30.3% and 30.2%, while the unconstrained R^2 is 31.4%. Hence, these factors supply a theory-based combination of forwards and yields, that does not require estimation. Thus, they are potentially portable across datasets.[28]

I conclude that the model explains all three CP findings, and proposes new combinations of factors to predict the bond premium. These are "estimation-free" and might be useful empirically.

5.5 Corporate Spread, Government Debt and Inflation Risk

Consider the corporate spread, which is the difference between the yield on the corporate bonds issued by the safest corporations (such as AAA firms) and government bonds. The "corporate spread puzzle" is that the spread is too high compared to the historical rate of default (Huang and Huang 2003). It has a very natural explanation under the disaster view. It is during disasters (in bad states of the world) that very safe corporations will default. Hence, the risk premia on default risk will be very high. To explore quantitatively this effect, I consider the case of a constant severity of disasters.

Proposition 8 *(Corporate bond spread, disasters, and expected inflation) Consider a corporation i, call F_i the (nominal) recovery rate of its bond, and λ_i the default rate conditional on no disaster, the yield on debt is $y_i = R + \lambda_i - pE\left[B^{-\gamma}F^{\$}F_i\right]$. So, calling y_G the yield on government bonds, the corporate spread is:*

$$y_i - y_G = \lambda_i + pE\left[B^{-\gamma}F^{\$}(1 - F_i)\right]. \tag{37}$$

[28] Another interesting combination is $8CP'(a = 1, b = 2) - \frac{1}{2}CP'(a = 2, b = 1)$, which is $-2f(1) + 0.5f(2) + 3f(3) + 0.5f(4) - 2f(5)$, and is very close to what CP estimate.

In particular, when inflation is expected to be high during disasters (i.e. $F^{\$}$ is low, perhaps because current Debt / GDP is high), then (i) the spread $|y_i - y_j|$ between two nominal asset i, j, is low, and (ii) the yields on nominal assets is high.

The disaster risk premium is $\pi_i^D = y_i - y_G - \lambda_i$, the difference between the yield on corporate bonds and governance bonds minus the historical default rate of corporate bonds. The rare disaster model gives a macroeconomic foundation for Almeida and Philippon (2007)'s view that the corporate spread reflects the existence of bad states of the world. Almeida and Philippon (2007) allow an estimate of π_i^D as the difference between this risk-adjusted annualized probability of default and the historical one.[29] For instance, it yields π_i^D to be about 4.05% for a bond rated B (resp., 0.60% for a AAA bond). With $pE\left[B^{-\gamma}\right] = 0.19$, it means that the expected loss in a disaster is $4.05\%/0.19 = 21\%$ (resp. $0.60\%/0.19 = 3.2\%$ for a AAA bond). This is a moderate loss. We see how easily, though, a disaster model can rationalize the corporate spread.

Prediction (i) of Proposition 8 seems quite new. The intuition for it is the following. Suppose agents know that there will be hyperinflation in disasters, so that the real value of all nominal assets will be zero ($F^{\$} = 0$). Then, there is no difference in the risk premium between government bonds, AAA bonds or any nominal bond: their value will be wiped out during disasters. So, the part of their spread due to inflation risk is 0. More generally, a higher inflation risk lowers the spread between nominal bonds, because it reduces the values of all nominal bonds.

Prediction (i) provides an explanation for Krishnamurthy and Vissing-Jorgensen (2008)'s finding that when the debt/GDP ratio is high the AAA-Treasury and the BAA-AAA spreads are low (see their Fig. 5). The first AAA-Treasury can be explained by their favored interpretation of a liquidity demand for treasuries, but the BAA-AAA spread is harder to explain via liquidity. The disaster hypothesis offers an explanation for both. When Debt/GDP is high the temptation to default via inflation (should a risk occur) is high so $F^{\$}$ is low, thus nominal spreads are low.

Prediction (ii) of Proposition 8 allows one to think about the impact of the government Debt/GDP ratio. It is plausible that if the Debt/GDP ratio is high then if there is a disaster the government will sacrifice monetary rectitude so that j_t is high (that effect could be microfounded). This implies that when the Debt/GDP ratio (or the deficit / GDP) is high then long-term rates are high and the slope of the yield curve is steep (controlling for inflation and expectations about future inflation in normal times). Dai and Philippon (2006) present evidence consistent with that view. Likewise, say that an independent central bank has a more credible commitment not to increase inflation during disasters (j_t smaller). Then, real long term rates (e.g. nominal rates minus expected inflation) are

[29] I take Almeida and Philippon (AP)'s Table III, which is the 10-year risk-neutral and historical probability, and apply the transformation $-\ln(1-x)/10$ to obtain the annualized probability of default. I also add back the AAA-Treasury spread of 0.51%, to get the actual AAA-Treasury spread. This yields a disaster premium of: 0.60% (AAA bonds), 1.11% (AA), 1.71% (A), 2.57% (BBB), 3.06% (BB), and 4.05% (B). AP do not report standard errors.

lower and the yield curve is less steep. This effect works in an economy where Ricardian equivalence (Barro 1974) holds. Higher deficits increase long term rates not because they "crowd out" investment, but instead because they increase the government's temptation to inflate away the debt if there is a disaster. In such a case there is an inflation risk premium on nominal bonds.

6 Discussion and Extensions

6.1 Other Interpretations of the Model

The model can have at least two other interpretations.

A Tractable Laboratory for Macro-Finance Much of what was derived in the sections of stocks and bonds (but not on options) does not depend finely on the disaster hypothesis, as the following Proposition formalizes.

Proposition 9 *(Models generating same stock and bond prices as a disaster economy) Consider a model with stochastic discount factor* $\frac{M_{t+1}}{M_t} = e^{-R}\left(1+\varepsilon_{t+1}^{M}\right)$*, and a stock with dividend following* $\frac{D_{t+1}}{D_t} = e^{g_D}\left(1+\varepsilon_{t+1}^{D}\right)$*, where all* ε_{t+1}*'s have expected value 0 at time* t*. Call* $H_t = E_t\left[\varepsilon_{t+1}^{M}\varepsilon_{t+1}^{D}\right] = H_* + \widehat{H}_{t+1}$*, so that* $-H_t$ *is the risk premium on the dividend, and assume* $\widehat{H}_{t+1} = \frac{1+H_*}{1+H_t}e^{-\phi_H}\widehat{H}_t + \varepsilon_{t+1}^{H}$ *with* ε_{t+1}^{H} *uncorrelated with the innovations to* $\frac{M_{t+1}}{M_t}\frac{D_{t+1}}{D_t}$*. Then, Theorem 1 and Proposition 1 hold, except that the equity premium is* $-H_t$*, and the interest rate is* R*.*

Furthermore, suppose that inflation is $i_t = i_* + \widehat{i}_t$*, and follows* $\widehat{i}_{t+1} = \frac{1-i_*}{1-i_t}\left(e^{-\phi_i}\widehat{i}_t + \varepsilon_{t+1}^{i}\right)$*. Call* $E_t\left[\varepsilon_{t+1}^{M}\varepsilon_{t+1}^{i}\right] = \pi_* + \pi_t$*, the inflation risk premium, and assume* $\pi_{t+1} = \frac{1-i_*}{1-i_t}e^{-\phi_j}\pi_t + \varepsilon_{t+1}^{\pi}$*, with* $E_t\left[\varepsilon_{t+1}^{M}\varepsilon_{t+1}^{\pi}\right] = 0$*. Also, use the notation* $\pi_* = (1-i_*)\,\kappa\left(1-e^{-\phi_i}-\kappa\right)$*. Then, Theorem 2 on bond values (with* $H^{\$} = 0$*), and Propositions 2, 5-7 on bond predictability hold (except Eq. 21).*

Proposition 9 shows that in many models stocks and bonds will behave exactly as in a disaster economy (however, options or defaultable bonds will be different). The critical assumption on $\widehat{H}_t$ is that the equity premium is stochastic. The primary assumption on i_t asserts that there is a stochastic inflation risk premium. From there, all the results on pricing and predictability follow. Hence, other microfoundations (e.g. habits) can lead to exactly the same economic behavior. One advantage of a disaster economy is that there is a clean microfoundation, it meshes well with the rest of macroeconomics, and there is a horizon of direct testability (as data on behavior during disasters are collected). In the mean time, disaster analytics shed light on many models without disasters.

A Model of "Time-Varying Perception of Risk" The model admits a behavioral interpretation. While the exposition so far has focused on rational agents, the model's time variation in the probability and severity of crashes could reflect behavioral biases. This variation can be due to changing investor perception of risk, sentiment or risk appetite. Therefore, the model offers a coherent way to think about the joint behavior of sentiment and prices. Under that interpretation one does not need to use the traditional "macroeconomic consumption drop" explanation. One can interpret the bad events as "financial crashes" with overweighing of small probability events. Indeed, the basic arbitrage equation of the paper $P_t = D_t + E\left[M_{t+1}/M_t \cdot P_{t+1}\right]$, can be rewritten in the case of constant B

$$P_t = D_t + e^{-R}\left(1 - p_t\right) E_t\left[P_{t+1} \mid \text{No crash}\right] + e^{-R} p_t B^{-\gamma} \cdot E_t\left[P_{t+1} \mid \text{Crash}\right].$$

The above equation does not refer to consumption. The agents basically follow an expected value maximization except that the $B^{-\gamma}$ term increases the effective weight put on low probability events consistent with prospect theory (Kahneman and Tversky 1979).

Additionally, the model's structure allows us to calculate stocks prices given a stochastic path of future sentiment. Baker and Wurgler (2006, 2007) find that periods of high (resp. low) sentiment are followed by low (resp. high) returns. This is exactly the prediction the model generates when high resilience H_t is interpreted as optimistic investor sentiment. Baker and Wurgler also find that the sentiment effect is more pronounced in small firms. If small firms have a more volatile resilience H_t, hence a higher "sentiment beta," then their prices in the model are more sensitive to changes in sentiment.

6.2 Cross-Sectional Predictability: Value and Growth Stocks

This subsection explores the consequences of the disaster hypothesis for value and growth stocks. The discussion is somewhat speculative. If a disaster happens, individual stocks will fare differently.[30] Their dividend will change by F_t, the recovery rate. This dispersion of sensitivity of dividends to disasters leads to a dispersion of premia and prices in normal times. I propose that this is a potential way to think about value and growth stocks (Lakonishok, Shleifer, and Vishny 1994, Fama and French 1996). In this rare-disaster interpretation the value premium is compensation for "distress risk" (Fama and French 1996, Campbell, Hilscher, and Szilagyi 2008) due to the company's behavior during economy-wide disasters.

There is little evidence on the hypothesis that value stocks do worse than growth stocks during

[30] For instance, stocks with a lot of physical assets that might be destroyed, or stocks very reliant on external finance, might have a lower F.

disasters. Gourio (2008) presents some mixed evidence based on US data. On the other hand, preliminary work with Joachim Voth investigates Russia and Germany in the first part of the century, and finds that value stocks do worse in disasters.

Also, with the "time-varying perception of risk" (rational or irrational) interpretation of disasters we can use the model's analytics to investigate the impact of "perception of risk" on cross-sectional prices and predictability. Stocks with a low resilience are "risky," as they will perform poorly during disasters. As per Theorem 1, they have a low price/dividend ratio, i.e. they are value stocks. By Proposition 1, they have high returns – a compensation for their riskiness during disasters.[31]

Characteristics predict returns better than covariances Suppose that a stock's resilience is constant i.e. $\widehat{H}_t \equiv 0$. In a sample without disasters, covariances between stocks are due to covariances between cash flows D_t in normal times. Thus, the stock market betas will only reflect the "normal times" covariance in cash flows. But, risk premia are only due to the behavior in disasters. Hence, the "normal times" betas can have no relation with risk premia. "Characteristics," like the P/D ratio, will predict returns better than covariances.

However, there could be some spurious links if stocks with low H_* have higher cash-flow betas. One could conclude that a cash-flow beta commands a risk premium, but this is not because cash-flow betas cause a risk premium. It is only because stocks with high cash-flow beta happen to also be stocks that have a large loading on the disaster risk.[32]

With an auxiliary assumption, this reasoning can explain the appearance of a "value factor" such as the High Minus Low (HML) factor of Fama and French. Suppose that resiliences have a 1-factor structure: $\widehat{H}_{it} = \beta_i^H \widehat{H}_{Mt} + \widehat{h}_{it}$, where $\widehat{H}_{Mt}$ is a systematic (*m*arket-wide) part of the expected resilience of the asset, and $\widehat{h}_{it}$ is the idiosyncratic part. Consider two benchmarks. If for all stocks $\beta_i^H = 1$ (the "characteristics benchmark"), all dispersion in $\widehat{H}_i$ is idiosyncratic and characteristics (the P/D ratio of a stock) predict future returns but covariances do not. On the other hand, if for all stocks $\widehat{h}_{it} \equiv 0$ but the β_i^H vary across stocks, all expected returns are captured by a covariance model. In general, the reality will be in between and covariances and characteristics are both useful to predict future returns.

These thought experiments may help explain the somewhat contradictory findings in the debate about whether characteristics or covariances explain returns (Daniel and Titman 1997, Davis,

[31] Fama and French (2007) show that the value premium is essentially due (at a descriptive level) to "migration," i.e. mean-reversion in P/E ratio: a stock with high (resp. low) P/E ratio tends to see its P/E ratio go up (resp. down). In terms of the model, this means that H_* is relatively constant across stocks, while the fluctuations in $\widehat{H}_t$ drive the value premium.

[32] Various authors (Julliard and Parker 2005, Campbell and Vuolteenaho 2004, Hansen, Heaton and Li 2009) find that value stocks have higher long run cash flow betas. It is plausible that stocks that have high cash-flow betas in normal times also have high cash-flow betas in disasters, i.e. a low F_t and a low H_t. But it is their disaster-time covariance that creates a risk premium, not the normal-time covariance.

Fama and French 2000). When covariances badly measure the true risk as in a disaster model characteristics will often predict expected returns better than covariances.

The value spread forecasts the value premium In the model, the value spread forecasts the value premium as Cohen, Polk, and Vuolteenaho (2003) have found empirically. To see this in a simple context, consider a cross-section of stocks, with identical permanent resiliences H_*. Say that at time t the support of their resiliences $\widehat{H}_t$ is $[a_t, b_t]$ where $a_t < b_t$. Then form a "Value minus Growth" portfolio made of \$1 of the extreme value stocks (low P/D), which correspond to $\widehat{H}_t = a_t$, minus \$1 of the extreme growth stocks (high P/D), which correspond to $\widehat{H}_t = b_t$. By Proposition 1, this portfolio has an expected payoff of $b_t - a_t$. On the other hand, the "value spread" in the P/D is: $(b_t - a_t) / [(r_i + \phi_H) r_i]$. The value spread is a perfect predictor of expected returns of a "Value minus Growth" strategy.

6.3 Cross-Asset Implications of the Model

The model makes cross-asset predictions if we assume that the shocks to resilience are correlated across assets – for instance, that a shock that increases bond premia also increases stock premia. When risk premia are high (high intensity of disaster, low $\widehat{H}_t$, high π_t): the slope of the yield curve is high (the bond premium is high); the price multiples (e.g. price / dividend, price / earnings, market / book ratios) of stocks are low; "growth stocks" have low P/E ratios; the value spread (e.g. measured as the difference of average market / book ratio in the top quintile of its distribution minus the bottom quintile) is low; put prices are high; options-based indices of volatility (e.g., VIX) are high; and the corporate bond spread is high. Furthermore, on average in the *future* returns on bonds will be high, long term yields will fall, short term rates will rise; returns on stocks will be high; returns on a "long value, short growth" (HML) position, will be low; returns on puts will be low, VIX will fall and corporate bond returns will be higher than Treasury bond returns, as the corporate bond spread mean-reverts.

Hence, in principle, the above view allows us to extract from stocks, bonds, and options underlying "deeper fundamentals" about the economy using the closed forms in this paper. Though this is beyond the scope of this paper it is a tempting avenue for research.

6.4 The Model with Many Factors

The previous sections have derived the main economics of the variable severity of disasters model. They relied on one nominal and one real risk factor. This section shows how the model readily extends to many factors, including those with high- and low-frequency predictability of dividend growth or inflation.

6.4.1 Extensions for Stocks

Variable Trend Growth Rate of Dividends It is easy to add a predictable growth rate trend to the stock's dividend. Postulate:

$$\frac{D_{t+1}}{D_t} = e^{g_D}\left(1+\varepsilon_{t+1}^D\right)\left(1+\widehat{g}_t\right) \times \begin{cases} 1 & \text{if there is no disaster at } t+1 \\ F_t & \text{if there is a disaster at } t+1 \end{cases}$$

where $\widehat{g}_t$ is the deviation of the growth rate from trend and follows a LG-twisted process: $E_t \widehat{g}_{t+1} = \frac{1+H_*}{(1+\widehat{g}_t)(1+H_t)} e^{-\phi_g} \widehat{g}_t$. Calling $\widehat{h}_t = (H_t - H_*)\left(1+\widehat{g}_t\right)/\left(1+H_*\right)$, postulate that $E_t \widehat{h}_{t+1} = \frac{1+H_*}{(1+\widehat{g}_t)(1+H_t)} e^{-\phi_H} \widehat{h}_t$.[33]

Proposition 10 *(Stock price with time-varying risk premium and time-varying growth rate of dividends) The price of stock i in the model with stochastic resilience $\widehat{H}_t$ and stochastic growth rate of dividend $\widehat{g}_t$ is in the limit of small time intervals:*

$$P_t = \frac{D_t}{r_i}\left(1 + \frac{\widehat{H}_t}{r_i+\phi_H} + \frac{\widehat{g}_t}{r_i+\phi_g}\right). \tag{38}$$

The expected return on the stock conditional on no disaster, is still $R_t^e = R - h_* - \widehat{H}_t$.

Eq. 38 nests the three main sources of variations of stock prices in a simple and natural way. Stock prices can increase because the level of dividends increases (D_t), because the expected future growth rate of dividends increases ($\widehat{g}_t$), or because the equity premium decreases ($\widehat{H}_t$). The growth and discount factors ($\widehat{g}_t$, $\widehat{H}_t$) enter linearly, weighted by their duration (e.g., $1/\left(r_i+\phi_H\right)$) which depends on the speed of mean-reversion of each process (parametrized by ϕ_π, ϕ_g) and the effective discount rate, r_i. The price is independent of the correlation between the instantaneous innovations in $\widehat{g}_t$ and $\widehat{H}_t$, as is typical of LG processes.

Stocks: Many Factors To have several factors for the growth rate and the discount factor, postulate: $\widehat{g}_t = \sum_{k=1}^{N_g} \widehat{g}_{k,t}$, $\widehat{h}_t = \sum_{k=1}^{N_H} \widehat{h}_{k,t}$, $E_t \widehat{g}_{k,t+1} = \frac{1+H_*}{(1+\widehat{g}_t)(1+H_t)} e^{-\phi_{g,k}} \widehat{g}_{k,t}$, and $E_t \widehat{h}_{t+1} = \frac{1+H_*}{(1+\widehat{g}_t)(1+H_t)} e^{-\phi_{H,k}} \widehat{h}_{k,t}$. For instance, the growth rates could correspond to different frequencies, i.e. a long run frequency (for low $\phi_{g,k}$) and a business cycle frequency (for a high $\phi_{g,k}$).

Proposition 11 *(Stock prices with time-varying risk premium and time-varying growth rates of dividends with an arbitrary number of factors.) The price of stock i in the model with stochastic resilience $\widehat{h}_t = \sum_{k=1}^{N_H} \widehat{h}_{k,t}$ and stochastic growth rate of dividend $\widehat{g}_t = \sum_{k=1}^{N_g} \widehat{g}_{k,t}$ is in the limit of*

[33] In continuous time, we have: $\widehat{h}_t = H_t - H_*$, $E_t d\widehat{g}_t = -\left(\phi_g + \widehat{g}_t + \widehat{h}_t\right)\widehat{g}_t dt$, $E_t \widehat{h}_t = -\left(\phi_H + \widehat{g}_t + \widehat{h}_t\right)\widehat{h}_t dt$.

small time intervals:

$$P_t = \frac{D_t}{r_i}\left(1 + \sum_{k=1}^{N_H} \frac{\widehat{h}_{k,t}}{r_i + \phi_{H,k}} + \sum_{k=1}^{N_g} \frac{\widehat{g}_{k,t}}{r_i + \phi_{g,k}}\right) \tag{39}$$

and the expected return on the stock, conditional on no disasters, is still: $R_t^e = R - h_* - \widehat{h}_t$.

Formula (39) is very versatile, and could be applied to a large number of cases.

6.4.2 Extensions for Bonds

Bonds: Variation in the Short Term Real Rate To highlight the role of risk premia, the real short-term interest rate is held constant in the baseline model. Making it variable is easy. Postulate that consumption follows $C_t = \widetilde{C}_t C_t^*$ where C_t^* follows the process seen so far (Eq. 1) and $\widetilde{C}_t$ captures a deviation of consumption from trend: $\widetilde{C}_{t+1}/\widetilde{C}_t = (1 - R_t)^{-1/\gamma}$ where R_t follows a LG process, $E_t[R_{t+1}] = e^{-\phi_R} R_t / (1 + R_t)$ with innovations in R_t uncorrelated with disasters and innovations in inflation variables i_t, j_t. When the consumption growth rate is high R_t is high. The pricing kernel is $M_t = \widetilde{C}_t^{-\gamma} M_t^*$ where M_t^* was is given in Eq. 2.

Proposition 12 *(Bond prices in the extended model) The bond price in the extended model is:*

$$Z_t(T) = Z_t^*(T)\left(1 - \frac{1 - e^{-\phi_R T}}{1 - e^{-\phi_R}} R_t\right)$$

where $Z_t^*(T)$ *is the bond price derived earlier in Theorem 2.*

In the continuous time limit, the short term rate is $r_t = R - H^{\$} + i_t + R_t$. It now depends both on inflation i_t and the consumption growth factor R_t.

Bonds: Many Factors The bond model admits more factors. For instance assume that inflation is the sum of K components, $i_t = i_* + \sum_{k=1}^{K} \widehat{i}_{kt}$, which follow:

$$\begin{aligned}
\widehat{i}_{k,t+1} &= \frac{1 - i_*}{1 - i_t} \cdot \left(\rho_k \widehat{i}_{kt} + 1_{\{\text{Disaster at } t+1\}}\left(j_{k*} + \widehat{j}_{kt}\right)\right) + \varepsilon_{k,t+1} \text{ for } k = 1, ..., K \\
\widehat{j}_{k,t+1} &= \frac{1 - i_*}{1 - i_t} \rho_{kj} \widehat{j}_t + \varepsilon_{t+1}^j
\end{aligned}$$

with $\left(\varepsilon_{1t}, ..., \varepsilon_{Kt}, \varepsilon_{1t}^j, ..., \varepsilon_{Kt}^j\right)$ having mean 0 independently of whether or not there is a disaster at t. The k-th component mean-reverts with an autocorrelation ρ_k which allows us to model inflation as the sum of fast and slow components. If there is a disaster, the k-th component of inflation jumps by an amount $\widehat{j}_{k,t}$ which will mean-revert quickly if ρ_k is small. I state the bond price in the simple case where there is no average increase in inflation: $\forall k, j_{k*} = 0$.

Proposition 13 *(Bond prices with several factors) In the bond model with K factors the bond price is:*

$$Z_t(T) = \left(e^{-R}\left(1+H^{\$}\right)(1-i_*)\right)^T \left(1 - \sum_{k=1}^{K} \frac{1-\rho_k^T}{1-\rho_k} \frac{\widehat{i}_{kt}}{1-i_*} - \sum_{k=1}^{K} \frac{\frac{1-\rho_k^T}{1-\rho_k} - \frac{1-\rho_{kj}^T}{1-\rho_{kj}}}{\rho_k - \rho_{kj}} \frac{p_t E_t\left[B_{t+1}^{-\gamma}\right]\widehat{j}_{kt}}{1-i_*}\right).$$

The bond price decreases with each component of inflation and more persistent components have greater impact.

7 Conclusion

This paper presents a tractable way to handle a time-varying severity of rare disasters, demonstrates its impact on stock and bond prices, and shows its implications for time-varying risk premia and asset predictability. Many finance puzzles can be understood through the lens of the variable rare disasters model. On the other hand, the model does suffer from several limitations and suggests several questions for future research.

First of all, it would be useful to empirically examine the model's joint expression of the values of stocks, bonds and options. In this paper, I have only examined their behavior separately, relying on robust stylized facts from many decades of research. The present study suggests specifications for the joined, cross-asset patterns of predictability.

It would be useful to understand how investors update their estimates of resiliences. Risk premia seem to decrease after good news for the economy (Campbell and Cochrane 1999) and for individual firms (the growth firms effect). So, it seems that updating will involve resiliences increasing after good news about the fundamental values of the economy or about individual stocks. Modeling that would lead to a link between recent events, risk premia, and future predictability. Preliminary notes suggest that this modelling is easy, given the analytics put forth in this paper.

This model is a step toward a unified framework for various puzzles in economics and finance. A companion paper (Farhi and Gabaix 2008) suggests that various puzzles in international macroeconomics (including the forward premium puzzle and the excess volatility puzzle on exchange rates) can be accounted for in an international version of the variable rare disasters framework. Furthermore, in ongoing work, I show how to embed the rare disasters idea in a production economy in a way that does not change its business cycle properties, but changes its asset pricing properties. This is possible because the disaster framework uses the same iso-elastic utility as the rest of macroeconomics rather than habit or Epstein-Zin-Weil utilities. Thus, the rare disasters idea may bring us closer to the long-sought goal of a joint, tractable framework for macroeconomics and finance.

Appendix A. Some Results on Linearity-Generating Processes

Basic Results This paper uses the Linearity-Generating (LG) processes defined and analyzed in Gabaix (2009). This Appendix describes the main results used here. LG processes are given by $M_t D_t$, a pricing kernel M_t times a dividend D_t, and X_t, an n-dimensional vector of factors (that can be thought of as stationary). For instance, for bonds, the dividend is $D_t = 1$. Here, I review the discrete time LG process. By definition, a process $M_t D_t (1, X_t)$ is LG if and only if, for all $t = 0, 1, ...$:

$$E_t \left[\frac{M_{t+1} D_{t+1}}{M_t D_t} \right] = \alpha + \delta' X_t \tag{40}$$

$$E_t \left[\frac{M_{t+1} D_{t+1}}{M_t D_t} X_{t+1} \right] = \gamma + \Gamma X_t \tag{41}$$

Higher moments need not be specified. For instance, the distribution of the noise does not matter, which makes LG processes parsimonious. As a short-hand, $M_t D_t (1, X_t)$ is a LG process with generator $\Omega = \begin{pmatrix} \alpha & \delta' \\ \gamma & \Gamma \end{pmatrix}$.

Stock and bond prices obtain in closed form. The price of a stock $P_t = E_t \left[\sum_{s \geq t} M_s D_s \right] / M_t$ is, with I_n, the identity matrix of dimension n:

$$P_t = D_t \frac{1 + \delta' (I_n - \Gamma)^{-1} X_t}{1 - \alpha - \delta' (I_n - \Gamma)^{-1} \gamma}. \tag{42}$$

The price-dividend ratio of a "bond" or $Z_t (T) = E_t [M_{t+T} D_{t+T}] / (M_t D_t)$, is:

$$Z_t (T) = \begin{pmatrix} 1 & 0_n \end{pmatrix} \Omega^T \begin{pmatrix} 1 \\ X_t \end{pmatrix} \tag{43}$$

$$= \alpha^T + \delta' \frac{\alpha^T I_n - \Gamma^T}{\alpha I_n - \Gamma} X_t \text{ when } \gamma = 0 \tag{44}$$

To ensure that the process is well-behaved (hence will prevent prices from being negative), the volatility of the process has to go to zero near some boundary. Gabaix (2009) details these conditions which footnote 9 illustrates.

Calibrating the Variance Suppose X_t is a LG-twisted process centered at 0 and $dX_t = -(\phi + X_t) X_t dt + \sigma (X_t) dW_t$, where W_t is a standard Brownian motion. Assume that, because of economic considerations, the support of X_t is some $[X_{\min}, X_{\max}]$, with $-\phi < X_{\min} < 0 < X_{\max}$. The following variance process makes that possible:

$$\sigma^2(X) = 2K(1 - X/X_{\min})^2 (1 - X/X_{\max})^2 \tag{45}$$

with $K > 0$. For the steady-state distribution to have a unimodal shape, it is useful to take $K \leq 0.2 \cdot \phi |X_{\min}| X_{\max}$.

The average variance of X_t is $\overline{\sigma}_X^2 = E[\sigma^2(X_t)] = \int_{X_{\min}}^{X_{\max}} \sigma(X)^2 p(X)\, dX$, where $p(X)$ is the steady state distribution of X_t. It can be calculated via the Forward Kolmogorov equation which yields $d \ln p(X)/dX = 2X(\phi + X)/\sigma^2(X) - d\ln \sigma^2(X)/dX$. Numerical simulations show that the process' volatility is fairly well-approximated by $\overline{\sigma}_X \simeq K^{1/2}\xi$, with $\xi = 1.3$. Also the standard deviation of X's steady state distribution is well-approximated by $(K/\phi)^{1/2}$. Asset prices often require that one analyzes the standard deviation of expressions like $\ln(1 + aX_t)$. Numerical analysis shows that the Taylor expansion approximation is a good one. The average volatility of $\ln(1 + aX_t)$ is $\overline{\sigma}_{\ln(1+aX_t)} \simeq aK^{1/2}\xi$, which numerical simulations also show to be a good approximation.

When the process is not centered at 0, one simply centers the values. For instance, in the calibration, the recovery rate of a stock F_t has support $[F_{\min}, F_{\max}] = [0, 1]$ centered around F_*. The probability and intensity of disasters (p and B) are constant. When we define $H_t = p(B^{-\gamma}F_t - 1)$ and the associated $H_{\min}$, $H_{\max}$, H_*. The associated centered process is $X_t = \widehat{H}_t = H_t - H_*$.

Appendix B. Longer Proofs

Proof of Lemma 1 The Euler equation, $1 = E_t[R_{t+1}M_{t+1}/M_t]$, gives:

$$1 = e^{-R}\left\{(1 - p_t)(1 + r_t^e) + p_t E_t\left[B_{t+1}^{-\gamma}\frac{P_{t+1}^{\#}}{P_t}\right]\right\}$$

hence (11).

Proof of Theorem 1 Following the general procedure for LG processes (Appendix A), I use (2), (3) and form:

$$\frac{M_{t+1}D_{t+1}}{M_t D_t} = e^{-R+g_D}\left(1 + \varepsilon_{t+1}^D\right) \times \begin{cases} 1 & \text{if there is no disaster at } t+1 \\ B_{t+1}^{-\gamma}F_{t+1} & \text{if there is a disaster at } t+1 \end{cases}$$

As the probability of disaster at $t+1$ is p_t, and $H_t \equiv p_t \left(E_t \left[B_{t+1}^{-\gamma} F_{t+1}\right] - 1\right)$,

$$
\begin{aligned}
E_t\left[\frac{M_{t+1}D_{t+1}}{M_t D_t}\right] &= e^{-R+g_D}\{\underbrace{(1-p_t)\cdot 1}_{\text{No disaster term}} + \underbrace{p_t \cdot E_t\left[B_{t+1}^{-\gamma}F_{t+1}\right]}_{\text{Disaster term}}\} \\
&= e^{-R+g_D}\left(1+H_t\right) = e^{-R+g_D}\left(1+H_* + \widehat{H}_t\right) = e^{-R+g_D+h_*}\left(1+e^{-h_*}\widehat{H}_t\right) \\
&= e^{-r_i}\left(1+e^{-h_*}\widehat{H}_t\right) \qquad (46)
\end{aligned}
$$

where I use the notations $h_* = \ln\left(1+H_*\right)$ and $r_i = R - g_D - h_*$. Next, as $\widehat{H}_{t+1}$ is independent of whether there is a disaster, and is uncorrelated with ε_{t+1}^D,

$$
\begin{aligned}
E_t\left[\frac{M_{t+1}D_{t+1}}{M_t D_t}\widehat{H}_{t+1}\right] &= E_t\left[\frac{M_{t+1}D_{t+1}}{M_t D_t}\right] E_t\left[\widehat{H}_{t+1}\right] = e^{-R+g_D}\left(1+H_t\right)\cdot\frac{1+H_*}{1+H_t}e^{-\phi_H}\widehat{H}_t \\
&= e^{-R+g_D+h_*-\phi_H}\widehat{H}_t = e^{-r_i-\phi_H}\widehat{H}_t \qquad (47)
\end{aligned}
$$

We see that in (5), the reason for the $1+H_t$ term in the denominator was to ensure that the above expression would remain linear in $\widehat{H}_t$.

There are two ways to conclude. The first way uses the results from Appendix A: Eq. 46 and 47 ensure that $M_t D_t \left(1, \widehat{H}_t\right)$ is a LG process with generator $\begin{pmatrix} e^{-r_i} & e^{-r_i-h_*} \\ 0 & e^{-r_i-\phi_H}\end{pmatrix}$. Eq. 42 gives the stock price (15). The second way (which is less rigorous, but does not require a knowledge of the results on LG processes) is to look for a solution of the type $P_t = D_t\left(a + b\widehat{H}_t\right)$ for some constants a and b. The price must satisfy: $P_t = D_t + E\left[M_{t+1}P_{t+1}/M_t\right]$, i.e., for all $\widehat{H}_t$,

$$
\begin{aligned}
a + b\widehat{H}_t = 1 + E_t\left[\frac{M_{t+1}D_{t+1}}{M_t D_t}\left(a+b\widehat{H}_{t+1}\right)\right] &= 1 + aE_t\left[\frac{M_{t+1}D_{t+1}}{M_t D_t}\right] + bE_t\left[\frac{M_{t+1}D_{t+1}}{M_t D_t}\widehat{H}_{t+1}\right] \\
= 1 + ae^{-r_i}\left(1+e^{-h_*}\widehat{H}_t\right) + be^{-r_i-\phi}\widehat{H}_t &= \left(1+ae^{-r_i}\right) + \left(ae^{-r_i-h_*} + be^{-r_i-\phi_H}\right)\widehat{H}_t.
\end{aligned}
$$

Solving for a and b, we get $a = 1 + ae^{-r_i}$, $b = ae^{-r_i-h_*} + be^{-r_i-\phi_H}$, and (15).

Proof of Theorem 2 I show the result in discrete time first, while the Online Appendix provides a proof in continuous time. The proof is simpler when $j_* = \kappa = 0$ and this is the best case to keep in mind in a first reading.

I call $\rho_i = e^{-\phi_i}$ and $\rho_j = e^{-\phi_j}$ and calculate the LG moments (Appendix A):

$$
E_t\left[\frac{M_{t+1}Q_{t+1}}{M_t Q_t}\right] = e^{-R}\left(1-i_t\right)\{\underbrace{(1-p_t)\cdot 1}_{\text{No disaster term}} + \underbrace{p_t\cdot E_t\left[B_{t+1}^{-\gamma}F_{t+1}^{\$}\right]}_{\text{Disaster term}}\} = e^{-R}\left(1+H^{\$}\right)\left(1-i_* - \widehat{i}_t\right). \qquad (48)
$$

$$\begin{aligned}
E_t\left[\frac{M_{t+1}Q_{t+1}}{M_tQ_t}\widehat{i}_{t+1}\right] &= e^{-R}\left(1-i_t\right)\{\underbrace{\left(1-p_t\right)E_t\left[\widehat{i}_{t+1}\mid \text{No dis. at } t+1\right]}_{\text{No disaster term}} \\
&\quad \underbrace{+p_t\cdot E_t\left[B_{t+1}^{-\gamma}F_{t+1}^{\$}\widehat{i}_{t+1}\mid \text{Dis. at } t+1\right]}_{\text{Disaster term}}\} \\
&= e^{-R}\left(1-i_t\right)\frac{1-i_*}{1-i_t}\{\left(1-p_t+p_tE_t\left[B_{t+1}^{-\gamma}F_{t+1}^{\$}\right]\right)\rho_i\widehat{i}_t+p_tE_t\left[B_{t+1}^{-\gamma}F_{t+1}^{\$}\right]\left(j_*+\widehat{j}_t\right)\} \\
&= e^{-R}\left(1+H^{\$}\right)\left(1-i_*\right)\left(\rho_i\widehat{i}_t+\frac{p_tE_t\left[B_{t+1}^{-\gamma}F_{t+1}^{\$}\right]}{1+H^{\$}}\left(j_*+\widehat{j}_t\right)\right) \\
&= e^{-R}\left(1+H^{\$}\right)\left(1-i_*\right)\left(\rho_i\widehat{i}_t+\left(1-i_*\right)\kappa\left(1-\rho_i-\kappa\right)+\pi_t\right).
\end{aligned}$$

using (9) and (10). This gives:

$$E_t\left[\frac{M_{t+1}Q_{t+1}}{M_tQ_t}\frac{\widehat{i}_{t+1}}{1-i_*}\right]=e^{-R}\left(1+H^{\$}\right)\left(1-i_*\right)\left(\kappa\left(1-\rho_i-\kappa\right)+\rho_i\frac{\widehat{i}_t}{1-i_*}+\frac{\pi_t}{1-i_*}\right). \tag{49}$$

Finally,

$$\begin{aligned}
E_t\left[\frac{M_{t+1}Q_{t+1}}{M_tQ_t}\widehat{j}_{t+1}\right] &= E_t\left[\frac{M_{t+1}Q_{t+1}}{M_tQ_t}\right]E_t\left[\widehat{j}_{t+1}\right]=e^{-R}\left(1+H^{\$}\right)\left(1-i_t\right)\cdot\frac{1-i_*}{1-i_t}\rho_j\widehat{j}_{t+1} \\
&= e^{-R}\left(1+H^{\$}\right)\left(1-i_*\right)\rho_j\widehat{j}_{t+1}
\end{aligned}$$

so that, using that $\pi_t/\left(1-i_*\right)$ is proportional to $\widehat{j}_t$ (Eq. 9),

$$E_t\left[\frac{M_{t+1}Q_{t+1}}{M_tQ_t}\frac{\pi_{t+1}}{1-i_*}\right]=e^{-R}\left(1+H^{\$}\right)\left(1-i_*\right)\rho_j\frac{\pi_{t+1}}{1-i_*}. \tag{50}$$

Hence $M_tQ_t\left(1,\frac{\widehat{i}_t}{1-i_*},\frac{\pi_t}{1-i_*}\right)$ is a LG process, with generator:

$$\Omega=e^{-R}\left(1+H^{\$}\right)\left(1-i_*\right)\begin{pmatrix}1 & -1 & 0\\ \kappa\left(1-\rho_i-\kappa\right) & \rho_i & 1\\ 0 & 0 & \rho_\pi\end{pmatrix}.$$

Eq. 43 gives the bond price, $Z_t\left(T\right)=\left(1,0,0\right)\Omega^T\left(1,\frac{\widehat{i}_t}{1-i_*},\frac{\pi_t}{1-i_*}\right)'$, which concludes the derivation of (18) when $\kappa=0$.

When $\kappa\neq 0$, one more step is needed. The eigenvalues of $\begin{pmatrix}1 & -1 & 0\\ \kappa\left(1-\rho_i-\kappa\right) & \rho_i & 1\\ 0 & 0 & \rho_\pi\end{pmatrix}$ are

$\{1-\kappa, \rho_i+\kappa, \rho_\pi\}$. It is convenient to factorize by $1-\kappa$, hence to define:

$$\widetilde{\rho}_i = (\rho_i+\kappa)/(1-\kappa), \qquad \widetilde{\rho}_\pi = \rho_\pi/(1-\kappa) \tag{51}$$

which are the discrete time analogues of the continuous time mean reversion speeds $\psi_i \equiv \phi_i - 2\kappa$ and $\psi_\pi \equiv \phi_\pi - \kappa$. Calculating Ω^T (diagonalizing Ω by hand or via symbolic calculation software) gives the bond price:

$$\begin{aligned} Z_t(T) &= \left(e^{-R}\left(1+H^{\$}\right)(1-i_*)(1-\kappa)\right)^T \times \\ & \quad \{1-\frac{1}{1-\kappa}\frac{1-\widetilde{\rho}_i^T}{1-\widetilde{\rho}_i}\left(\frac{\widehat{i}_t}{1-i_*}-\kappa\right)-\frac{1}{(1-\kappa)^2}\frac{\frac{1-\widetilde{\rho}_i^T}{1-\widetilde{\rho}_i}-\frac{1-\widetilde{\rho}_\pi^T}{1-\widetilde{\rho}_\pi}}{\widetilde{\rho}_i-\widetilde{\rho}_\pi}\frac{\pi_t}{1-i_*}\} \end{aligned} \tag{52}$$

Taking the continuous time limit yields (18). The corresponding value of the yield $y_t(T) = -(\ln Z_t(T))/T$ is:

$$\begin{aligned} y_t(T) &= R-H^{\$}+i_{**}+\frac{1-e^{-\psi_i T}}{\psi_i T}(i_t-i_{**})+\frac{\frac{1-e^{-\psi_i T}}{\psi_i}-\frac{1-e^{-\psi_\pi T}}{\psi_i}}{(\psi_\pi-\psi_i)T}\pi_t+O(i_t-i_{**},\pi_t)^2 &(53)\\ &= R-H^{\$}+i_{**}+\left(1-\frac{\psi_i T}{2}+\frac{\psi_i^2T^2}{6}\right)(i_t-i_{**})+\left(\frac{T}{2}-\frac{\psi_i+\psi_\pi}{6}T^2\right)\pi_t &(54)\\ &\quad +O\left(T^3\right)+O(i_t-i_{**},\pi_t)^2 \\ &= R-H^{\$}+i_t+\left(\kappa\left(\phi_i-\kappa\right)+\pi_t-\phi_i\left(i_t-i_*\right)\right)\frac{T}{2}. &(55) \end{aligned}$$

For instance, (55) means that when inflation is at baseline ($i_t = i_*$) the slope of the yield curve is half the bond maturity, times the bond risk premium, $\kappa(\phi_i-\kappa)+\pi_t$: the first term is the permanent part of the bond risk premium, the second is its variable part.

Proof of Proposition 1 If a disaster occurs, dividends are multiplied by B_tF_t. As $\widehat{H}_t$ does not change, $P_t^{\#}/P_t = F_t$. So returns are, by Eq. 12,

$$r_t^e = R + p_t\left(1-E_t\left[B_t^{-\gamma}F_t\right]\right) = R - H_t = R - H_* - \widehat{H}_t.$$

Proof of Proposition 2 After a disaster π_t does not change but i_t jumps to $i_t + j_* + \widehat{j}_t$. The bond holder suffers a capital loss: $V_t - V_t^{\#} = e^{-\left(R-H^{\$}+i_{**}\right)T}\cdot\frac{1-e^{-\psi_i T}}{\psi_i}\left(j_*+\widehat{j}_t\right)$. Lemma 1 gives the risk premia, using $p_tE_t\left[B_{t+1}^{-\gamma}F_{t+1}^{\$}\left(j_*+\widehat{j}_t\right)\right] = \kappa(\phi_i-\kappa)+\pi_t = \kappa(\psi_i+\kappa)+\pi_t$.

Proof of Proposition 3 Proposition 1 gives the expected returns over a short horizon T to be $r_{t,T}^e = (R-H_t)T$. Eq. 16 implies that the right-hand side of (25) is to the leading order

$\ln (D/P)_t = -\ln r_i - \widehat{H}_t / (r_i + \phi_H)$. So the regression is to a first order:

$$r^e_{t,T} = \left(R - H_* - \widehat{H}_t\right) T = \text{constant} - \beta_T \frac{\widehat{H}_t}{r_i + \phi_H} + \text{noise}.$$

Thus by inspection, $\beta_T = (r_i + \phi_H) T$. By the same reasoning regression (26) is $r^e_{t,T} = \left(R - H_* - \widehat{H}_t\right) T =$ constant$-\beta'_T \frac{r_i \widehat{H}_t}{r_i+\phi_H}$+noise, so $\beta'_T = (r_i + \phi_H) T / r_i$.

Proof of Proposition 4 $V_t = V_t^{ND} + V_t^D$ with:

$$\begin{aligned} V_t^{ND} &= (1-p_t) E_t \left[e^{-R} \left(K - \frac{P_{t+1}}{P_t} \right)^+ \mid \text{No disaster} \right] = (1-p_t) e^{-R} E_t \left[\left(K - e^{\mu + \sigma u_{t+1} - \sigma^2/2} \right)^+ \right] \\ V_t^D &= p_t E_t \left[e^{-R} B_{t+1}^{-\gamma} \left(K - \frac{P_{t+1}}{P_t} \right)^+ \mid \text{Disaster} \right] = p_t e^{-R} E_t \left[B_{t+1}^{-\gamma} \left(K - e^{\mu} F_{t+1} \right)^+ \right] \end{aligned}$$

Recall that the Black-Scholes value of a put with maturity 1 is: $E_t \left[e^{-r} \left(K - e^{r + \sigma u_{t+1} - \sigma^2/2} \right)^+ \right] = V^{BS}_{Put} (K e^{-r}, \sigma)$. Hence, the first term is:

$$\begin{aligned} (1-p_t) e^{-R} E_t \left[\left(K - e^{\mu + \sigma u_{t+1} - \sigma^2/2} \right)^+ \right] &= (1-p_t) e^{-R+\mu} E_t \left[\left(K e^{-\mu} - e^{\sigma u_{t+1} - \sigma^2/2} \right)^+ \right] \\ &= (1-p_t) e^{-R+\mu} V^{BS}_{Put} \left(K e^{-\mu}, \sigma \right). \end{aligned}$$

Proof of Proposition 5 The Fama-Bliss regression (30) yields:

$$\beta_T = \frac{cov \left(R^e(T) - R^e(0), f_t(T) - f_t(0) \right)}{var \left(f_t(T) - f_t(0) \right)}.$$

Eq. 19 and 23 give:

$$\begin{aligned} f_t(T) - f_t(0) &= \left(e^{-\psi_i T} - 1 \right) i_t + \frac{e^{-\psi_i T} - e^{-\psi_\pi T}}{\psi_\pi - \psi_i} \pi_t + a_T + O(i_t, \pi_t)^2 \\ R^e(T) - R^e(0) &= \frac{1 - e^{-\psi_i T}}{\psi_i} \pi_t + O(i_t, \pi_t)^2 \end{aligned}$$

where a_T is a constant. So up to $O(var(\pi_t), var(i_t))^{3/2}$ terms in the numerator and the denominator,

$$\beta_T = \frac{\frac{e^{-\psi_i T} - e^{-\psi_\pi T}}{\psi_\pi - \psi_i} \frac{1 - e^{-\psi_i T}}{\psi_i} var(\pi_t)}{var \left(\left(e^{-\psi_i T} - 1 \right) i_t + \frac{e^{-\psi_i T} - e^{-\psi_\pi T}}{\psi_\pi - \psi_i} \pi_t \right)}, \tag{56}$$

which implies that: $\lim_{T\to\infty}\beta_T = 0$, $\lim_{T\to 0}\beta_T = var(\pi_t)/var(\psi_i i_t + \pi_t)$, and (31).

Proof of Proposition 6 This proof is in the limit of $\sigma_i \to 0$, $i_t = 0$, $\kappa \to 0$, and $\Delta t \to 0$. Eq. 53 gives: $y_t(T) = a + b(T)\pi_t$, with $b(T) = \frac{\frac{1-e^{-\psi_i T}}{\psi_i} - \frac{1-e^{-\psi_\pi T}}{\psi_i}}{(\psi_\pi - \psi_i)T} = \frac{T}{2} - \frac{\psi_i + \psi_\pi}{6}T^2 + O(T^3)$. Hence:

$$\frac{y_{t+\Delta t}(T-\Delta t) - y_t(T)}{\Delta t} = E_t[dy_t(T)]/dt - \partial y_t(T)/\partial T = (-\phi_\pi b(T) - b'(T))\pi_t$$

As $(y_t(T) - r_t)/T = b(T)\pi_t/T$, $-\beta = \frac{\phi_\pi b(T) + b'(T)}{b(T)/T}$, i.e.

$$\beta = \frac{-Tb'(T)}{b(T)} - \phi_\pi T \tag{57}$$

so that $\beta = -1 - \frac{2\psi_\pi - \psi_i}{3}T + O(T^2)$ when $T \to 0$ and $\beta = -\psi_\pi T + o(T)$ when $T \to \infty$. The reasoning in the text of the paper comes from the fact that, for small T, $E_t[dy_t(T)]/dt = -\frac{\phi_\pi T}{2}\pi_t$, $-\partial y_t(T)/\partial T = \left(-\frac{1}{2} + O(T)\right)\pi_t$, so $\frac{y_{t+\Delta t}(T-\Delta t) - y_t(T)}{\Delta t} \simeq -\frac{\partial y_t(T)}{\partial T}$.

Proof of Proposition 7 From (23) and (53) up to third order terms $CP'_t = \left(\psi_i + \psi_j\right)\pi_t = CP''_t$. The leading inflation term is $-\psi_i^2 i_t$, a third order term.

Proof of Proposition 8 The Euler equation is:

$$1 = e^{-R}(1+y_i)\left[(1-p)(1-\lambda_i) + pE\left[B^{-\gamma}F^{\$}F_i\right]\right]$$

and the Proposition follows by taking the limit of small time intervals.

Proof of Proposition 9 With $Q_{t+1}/Q_t = 1 - i_t$, $M_t D_t\left(1, \widehat{H}_t\right)$ and $M_t Q_t\left(1, \frac{\widehat{i}_t}{1-i_*}, \frac{\pi_t}{1-i_*}\right)$ are both LG processes, with the same moments as in the disaster economy.

Proof of Proposition 10 $M_t D_t\left(1, \widehat{h}_t, \widehat{g}_t\right)$ is a LG process, so the PD ratio obtains from the basic results reviewed in Appendix A.

Proof of Proposition 11 $M_t D_t\left(1, \widehat{h}_{1t}, ..., \widehat{h}_{N_k}, \widehat{g}_{1,t}, ..., \widehat{g}_{N_g,t}\right)$ is a LG process, so the PD ratio obtains from the basic results reviewed in Appendix A.

Proof of Proposition 12 Because $\widetilde{C}_t^{-\gamma}(1, R_t)$ is a LG process, we have: $E_t\left[\frac{\widetilde{C}_{t+T}^{-\gamma}}{\widetilde{C}_t^{-\gamma}}\right] = 1 - \frac{1-e^{-\phi_R T}}{1-e^{-\phi_R}}R_t$. With Q_t the real value of money, the nominal bond price is: $Z_t = E_t\left[\frac{M_{t+T}Q_{t+T}}{M_t Q_{t+T}}\right] =$

$$E_t\left[\frac{M_t^* Q_{t+T}}{M_t^* Q_{t+T}}\right] E_t\left[\frac{\widetilde{C}_{t+T}^{-\gamma}}{\widetilde{C}_t^{-\gamma}}\right] = Z_t^*(T)\, E_t\left[\frac{\widetilde{C}_{t+T}^{-\gamma}}{\widetilde{C}_t^{-\gamma}}\right].$$

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